ASSESSMENT OF ESTIMATION TECHNIQUES OF SIMULTANEOUS EQUATION MODEL WITH MULTICOLLINEARITY PROBLEM UNDER NORMALLY AND UNIFORMLY DISTRIBUTED EXOGENOUS VARIABLES

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Abstract: The exogenous variables in Simultaneous Equation Model (SEM) may be related. This situation is referred to as multicollinearity. The estimators of SEM are not expected to give optimal result when there is multicollinearity. Consequently, this paper attempts to assess the performances of Ordinary Least Squares (OLS), Two Stage Least Squares (2SLS), Three Stage Least Squares (3SLS) and Full Information Maximum Likelihood (FIML) estimators in the presence of multicollinearity in both exact and over identified equations. Finite properties of estimators' criteria namely bias, variance, mean Absolute error and mean squared error were used as criteria to compare the estimators. Results show that the performances of the estimator under normal distributed exogenous variables are not always the same with the performances of the estimator under uniformly distributed exogenous variables. In low multicollinearity level, the OLS estimator is the most efficient except in over identified equation when the sample size is very large (n≥250). At these instances 2SLS is most efficient. With moderate multicollinearity level, OLS estimator is generally best in exact and over identified equation except when sample size is very large (n = 500) with normally distributed exogenous variables and when sample size is moderate (n = 50 and100) with uniformly distributed exogenous variables, at these instances, 2SLS or 3SLS estimators is most efficient.

Keywords: Multicollinearity, normally distributed exogenous variables, uniformly distributed exogenous variables, identification status

Introduction

A simultaneous equation system is a regression equation system where two types of variables (the endogenous, the predetermined or exogenous variable) appear with disturbance terms. Schmidt (2005) defined simultaneous equation as the process of modeling more than one equation at a time; a multi-equation modeling.

In a multi- equation model, the dependent variable Y appears as endogenous variable in one equation and as explanatory variable in another equation of the model. The X variable appears as the explanatory variable in the equations. This creates problems of equation identification, multicollinearity and choice of estimation techniques among others.

In the simultaneous, the problem of multicollinearity may still exist as the exogenous variables in the model can be related, and relationship between endogenous variables in each equation is also not impossible. When a single equation is embedded in a system of simultaneous equations, at least one of the right hand side variables will be endogenous and consequently the error terms of the equation will be correlated. This follows from the simultaneity of the system which means that all equations are jointly determined.

The problem of multicollinearity in simultaneous equations model has been examined and studied by several authors including Olubusoye (2001), Oduntan(2004), Johnson, Oyejola and Ayinde (2010), Agunbiade and Iyaniwura (2010), Ayinde, Johnson and Oyejola (2011), Agunbiade (2011), Agunbiade (2012), Alabi and Oyejola (2015) and Alabi (2016). These authors examined the performance of the Ordinary Least Squares (OLS), Indirect Least Squares(ILS), Two Stage Least Squares (2SLS), Three Stage Least Squares (3SLS), Least Information Maximum Likelihood (LIML) and Full Information Maximum Likelihood (FIML) under diverse specification of multicollinearity.

Identification problem which creates estimation problems has the same features with multicollinearity Koutsoyainnis (1977). Correlation between the pairs of exogenous or independent variables is an important problem in econometrics especially in single equation estimation. The impact of multicollinearity is less serious when attention is focused on predicting or forecasting values of the endogenous variables than when the analyst is interested in estimating the parameters Oduntan (2004).

Econometric variables are often non-negative and can exhibit violation against the normality assumption of classical models which inevitably influences the performances of the estimation techniques. This study therefore examines the performances of four (4) common estimation techniques namely; Ordinary Least squares (OLS), Two-stage Least squares (2SLS), Three-stage Least squares (3SLS) and Full Information Maximum Likelihood Estimators under both normally and uniformly distributed exogenous variables for different equation identification status. Here it is assumed that there is no form of correlation between the error terms in the simultaneous equation model.

Most economic data are often positive Kmenta *et al.* (1967) and correlated Chatterjee *et al.* (2012). However, various works done in the recent time especially on correlation studies on simultaneous equation model have being with normally distributed exogenous variables, exhibiting both positive and negative values Johnson *et al.* (2010), Ayinde *et al.* (2011a) and Ayinde *et al.* (2011b).

In a single regression model, multicollinearity has been identified to contribute serious threat to the efficiency of the Ordinary Least Square (OLS) estimation method of the linear regression model. Among the suggested methods of handling the problem is the use of Simultaneous Equation Model. Simultaneous equation model arises when the exogenous variables in single equation correlated with the error term.

The problem of multicollinearity in simultaneous equations model has been examined and studied by several authors. Some of the authors that had worked on this are: Olubusoye (2001), Oduntan (2004), Johnson, Oyejola and Ayinde (2010), Agunbiade and Iyaniwura (2010), Ayinde *et al.* (2011), Agunbiade (2011), Agunbiade (2012), Alabi and Oyejola (2015) and Alabi (2016). These authors examined the performance of the Ordinary Least Squares (OLS), Indirect Least Squares (ILS), Two Stage Least Squares (2SLS), Three Stage Least Squares (3SLS), Least Information Maximum Likelihood (LIML) and Full Information Maximum Likelihood (FIML) under diverse specification of multicollinearity. Their assessment was done under the normally distributed exogenous variables. But the work of Alabi and Oyejola (2015) was on the Assessments of some simultaneous equation estimation techniques with normally and uniformly distributed exogenous variables under different identification status of SEM when there is no correlation of any form in the model. Identification problem which creates estimation problems has the same features with multicollinearity Koutsoyainnis (1977). The impact of multicollinearity is less serious when attention is focused on predicting or forecasting values of the endogenous variables than when the analyst is interested in estimating the parameters Oduntan (2004).

Materials and Methods

The methodology followed in this research work is as follows:

The model and its description

Consider the simultaneous Equation model of the form

$$y_{t1} = \beta_{12}y_{t2} + \beta_{13}y_{t3} + \beta_{14}y_{t4} + \gamma_{14}x_{t4} + u_{t1} \quad \text{(i)}$$
$$y_{t2} = \beta_{23}y_{t3} + \gamma_{21}x_{t1} + \gamma_{23}x_{t3} + u_{t2} \quad \text{(ii)}$$
$$y_{t3} = \beta_{31}y_{t1} + \beta_{34}y_{t4} + \gamma_{31}x_{t1} + \gamma_{32}x_{t2} + u_{t3} \quad \text{(iii)}$$
$$y_{t4} = \beta_{41}y_{t1} + \beta_{42}y_{t2} + \gamma_{42}x_{t2} + u_{t4} \quad \text{(iv)}$$

(1)

where y_{it} is an endogenous variable, $i = 1,2,3,4$; x_{t1}, x_{t2}, x_{t3} and x_{4t} are the exogenous variables

$$\begin{bmatrix} u_{t1} \\ u_{t2} \\ u_{t3} \\ u_{t4} \end{bmatrix} \sim N(0, \Sigma), \text{ where } t=1,2,3,\dots,n$$

and $\beta_{12}, \beta_{13}, \beta_{14}, \beta_{23}, \beta_{31}, \beta_{34}, \beta_{41}, \beta_{42}, \gamma_{14}, \gamma_{21}, \gamma_{23}, \gamma_{31}\ \gamma_{32}$ and γ_{42} are the structural parameters of the model. Equations (i) and (iii) are exact identified while equations (ii) and (iv) are over identified by both order and rank condition.

For the simulation study, equation (1) was expressed as follows:

$$y_{t1} - \beta_{12}y_{t2} - \beta_{13}y_{t3} - \beta_{14}y_{t4} + 0x_{t1} + 0x_{t2} + 0x_{t3} - \gamma_{14}x_{t4} = u_{t1} \quad \text{(i)}$$
$$0y_{t1} + y_{t2} - \beta_{23}y_{t3} + 0y_{t4} - \gamma_{21}x_{t1} + 0x_{t2} - \gamma_{23}x_{t3} + 0x_{t4} = u_{t2} \quad \text{(ii)}$$
$$-\beta_{31}y_{t1} + 0y_{t2} + y_{t3} - \beta_{34}y_{t4} - \gamma_{31}x_{t1} - \gamma_{32}x_{t2} + 0x_{t3} + 0x_{t4} = u_{t3} \quad \text{(iii)}$$
$$-\beta_{41}y_{t1} - \beta_{42}y_{t2} + 0y_{t3} + y_{t4} + 0x_{t1} - \gamma_{42}x_{t2} + 0x_{3t} + 0x_{t4} = u_{t4} \quad \text{(iv)}$$

(2)

This can be written in matrix form as:

$$\beta y_t + \Gamma x_t = u_t \quad (3)$$

where

$$\beta = \begin{bmatrix} 1 & -\beta_{12} & -\beta_{13} & -\beta_{14} \\ 0 & 1 & -\beta_{23} & 0 \\ -\beta_{31} & 0 & 1 & -\beta_{34} \\ -\beta_{41} & -\beta_{42} & 0 & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0 & 0 & 0 & -\gamma_{14} \\ -\gamma_{21} & 0 & -\gamma_{23} & 0 \\ -\gamma_{31} & -\gamma_{32} & 0 & 0 \\ 0 & -\gamma_{42} & 0 & 0 \end{bmatrix}, \quad y_t = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix},$$

$$x_t = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \text{ and } u_t = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

Now, from (3),

$$\beta^{-1}\beta y_t = \beta^{-1}\Gamma x_t = \beta^{-1}u_{t\,4} \quad (4)$$

Data generation of exogenous variables

The generation was done as follows:

Normally distributed exogenous variables

Multicollinearity is built into the exogenous variables in equation (1) using the equation provided by Ayinde (2007). The equations are given as:

$$X_1 = \mu_1 + \sigma_1 z_1$$
$$X_2 = \mu_2 + \rho_{12}\sigma_2 z_1 + (\sqrt{m_{22}})z_2$$
$$X_3 = \mu_3 + \rho_{13}\sigma_3 z_1 + \frac{m_{23}}{\sqrt{m_{22}}} z_2 + \left(\sqrt{n_{33}}\right)z_3 \qquad (5)$$
$$X_4 = \mu_3 + \rho_{14}\sigma_4 z_1 + \frac{m_{24}}{\sqrt{m_{22}}} z_2 + \frac{n_{34}}{\sqrt{n_{33}}} z_3 + \sqrt{O}_{44} z_4$$

Where: $m_{22} = \sigma_2^2\left(1-\rho_{12}^2\right)$, $m_{23} = \sigma_2\sigma_3\left(\rho_{23}-\rho_{12}\rho_{13}\right)$,

$m_{24} = \sigma_2\sigma_4\left(\rho_{24}-\rho_{12}\rho_{14}\right)$, $m_{33} = \sigma_3^2\left(1-\rho_{13}^2\right)$, $m_{44} = \sigma_4^2\left(1-\rho_{14}^2\right)$,

$O_{44} = n_{44} - \frac{n_{34}^2}{n_{33}}$, $n_{44} = m_{44} - \frac{m_{24}^2}{m_{22}}$, $n_{34} = m_{34} - \frac{m_{23}m_{24}}{m_{22}}$, $n_{33} = m_{33} - \frac{m_{23}^2}{m_{22}}$,

and $Z_i \sim N(0,1)$ $i = 1,2,3,4$ and $\left|\rho_{ij}\right|$<1 is the value of correlation between the two variables i and j (Ayinde , 2007).

The exogenous were generated to be normal with mean zero and variance unity i.e. $x_i \sim N(0,1)$ $i = 1,2,3,4$ and $\left|\rho_{ij}\right|$<1 is the value of correlation between the two variables i and j.

Also, $X_i \sim$ N(0,1) for i=1, 2, 3, 4 (Ayinde, 2007).

In this study, $\rho_{12} = \rho_{13} = \rho_{14} = \rho_{23} = \rho_{24} = \rho_{34} = \rho =$ 0.3, 0.6, 0.9 and 0.99 was adopted as a situation with presence of multicollinearity between the exogenous variables in the model.

Correlated uniformly distributed variables

Using the generated correlated normally distributed exogenous variables $X_i \sim$ N(0,1), i=1, 2, 3, 4; we further utilized the properties of random variables that cumulative distribution function of normal distribution produces U(0,1) without affecting the correlation among the variables to generate correlated uniformly distributed exogenous variables, $X_i \sim$ U(0,1), i=1, 2, 3, 4 (Schumann, 2009).

Uniformly distributed exogenous variables

Using the generated normally distributed exogenous variables above, $X_i \sim$ N(0,1), i=1, 2, 3, 4; we further utilizes the properties of random variables that cumulative distribution function of normal distribution produces U(0,1) without affecting the correlation among the variables to generate correlated uniformly distributed exogenous variables, $X_i \sim$ U(0,1), i=1, 2, 3, 4; Schuman(2009).

Generation of correlated error terms data

Equation provided by Ayinde, (2007) was modified when the mean of the error terms are zero and variance is unit (1).

The correlation values are: $\lambda_{12} = \lambda_{13} = \lambda_{14} = \lambda_{23} = \lambda_{24} = \lambda_{34} = \lambda$ and $\left|\lambda_{ij}\right|$<1 is the value of correlation between the two error terms i and j.

Also, $\left|\lambda_{ij}\right|$<1 is the value of correlation between the two error terms i and j. In this study, $\lambda_{12} = \lambda_{13} = \lambda_{14} = \lambda_{23} = \lambda_{24} = \lambda_{34} = \lambda$ =0, was adopted as the situation with no correlation of any form between the error terms in the model.

Methods of generating endogenous variables Data:

Equation (4) was used to generate the endogenous variables by taking the true value of the parameters as:

$$\beta = \begin{bmatrix} 1 & -\beta_{12} & -\beta_{13} & -\beta_{14} \\ 0 & 1 & -\beta_{23} & 0 \\ -\beta_{31} & 0 & 1 & -\beta_{34} \\ -\beta_{41} & -\beta_{42} & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3.2 & -0.2 & -1.2 \\ 0 & 1 & -3.8 & 0 \\ -1.6 & 0 & 1 & -1.0 \\ -2.8 & -2.2 & 0 & 1 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} 0 & 0 & 0 & -\gamma_{14} \\ -\gamma_{21} & 0 & -\gamma_{23} & 0 \\ -\gamma_{31} & -\gamma_{32} & 0 & 0 \\ 0 & -\gamma_{42} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -0.8 \\ -3.0 & 0 & -1.3 & 0 \\ -1.5 & -0.6 & 0 & 0 \\ 0 & -2.6 & 0 & 0 \end{bmatrix}$$

Monte Carlo experiments were performed 5000 times for seven sample sizes (n=10, 20, 30, 50, 100, 250 and 500) when there is only multicollinearity between the exogenous variables of the model.
The two forms of exogenous variables are generated to follow normal and uniform distribution.

Criteria for assessment

To assess the performance of the estimators, the finite properties were used (Bias, Variance Mean Absolute error(MAE) and Mean Square Error(MSE)):
Mathematically, MSE is the addition of the Bias squared to the variance. For any estimator $\hat{\theta}_{ij}$ of θ_{ij} in model (1),

$$\hat{\theta}_{ij} = \frac{1}{R}\sum_{l=1}^{R}\hat{\theta}_{ijl}\ ,\quad l = 1, 2, \ldots, R\text{ , where R=5000}$$

$$\text{BIAS}\left(\hat{\theta}_{ij}\right) = \hat{\theta}_{ij} - \theta_{ij}\ ,$$

$$\text{Var}\left(\hat{\theta}_{ij}\right) = \text{MSE}\left(\hat{\theta}_{ij}\right) - \left[Bias\left(\hat{\theta}_{ij}\right)\right]^2$$

$$\text{MAE}\left(\hat{\theta}_{ij}\right) = \frac{1}{R}\sum_{l=1}^{R}\left|\hat{\theta}_{ijl} - \theta_{ij}\right|,\ \text{as a measure of dispersion.}$$

$$\text{MSE}\left(\hat{\theta}_{ij}\right) = \frac{1}{R}\sum_{l=1}^{R}\left(\hat{\theta}_{ijl} - \theta_{ij}\right)^2$$

In this study, $\hat{\theta}$ is used to represent the estimator of any of the parameters in model (1) under consideration.
The bias (closest to zero), variance, Mean absolute error and mean square error values were obtained for each of the parameters of the equation of the estimation method in all the sample sizes at all the levels of correlation values for both the normally distributed exogenous variables and uniformly distributed exogenous variables via Time series processor package programme, Time series processor (2005).

Summary of the steps taken to identify best estimator(s)

For the real simulated data, the following steps were taken to assess the performance of the estimators. At a particular specification of exogenous variable, sample size and multicollinearity level the following task was accomplished on information gathered for each criterion.

1. Value of each criterion was obtained for each of the parameters in the model via a TSP programme.
2. The estimators were ranked using each criterion; rank 1 for estimate closet to actual value of parameter (for bias) and smallest mean square error/variance.
3. The ranks of each estimator were summed over the parameters in each equation of the model.
4. The summed ranks in 3 were further added over each of the equation identification status.
5. Preferred or best estimator is determined in 4 for each sample size as the estimator with the least rank sum values.

Results and Discussion

In this section, the performances of the estimators are compared when there is presence of multicollinearity.

Bias examination of the model parameters

The summary of the performances of the estimators having summed the rank of bias over all the parameters in each equation was obtained for each selected sample size at each selected level of multicollinearity.
For each sample size, the summed ranks values were further added over the equations, and preferred estimator(s) under both exactly and over identification model with the two exogenous variables were obtained (Appendix 1) and the summary of the preferred estimators are given in Table 1.

Table 1: Summary of the preferred estimators) under the bias criterion when there is presence of multicollinearity alone in the model

ρ	N	Normal Exogenous Variables		Uniform Exogenous Variables	
		Exactly identified	Over identified	Exactly identified	Over – identified
	10	2SLS	3SLS	OLS	2SLS
0.3	20	OLS	2SLS\3SLS	OLS\2SLS	2SLS
	30	OLS	3SLS	OLS	2SLS
	50	2SLS	3SLS	OLS	2SLS
	100	2SLS	2SLS	OLS	2SLS\3SLS
	250	3SLS	2SLS	2SLS	2SLS
	500	3SLS	FIML	2SLS	FIML
0.6	10	OLS	2SLS	OLS	OSLS\2SLS
	20	2SLS	2SLS	OLS	2SLS
	30	OLS	2SLS\3SLS	2SLS	FIML
	50	2SLS	FIML	OLS	2SLS
	100	2SLS	3SLS	2SLS	2SLS
	250	2SLS	2SLS\3SLS	2SLS	FIML
	500	3SLS	FIML	2SLS	2SLS
0.9	10	OLS	2SLS	OLS\2SLS	2SLS
	20	OLS	3SLS\FIML	OLS	OLS\2SLS\3SLS
	30	OLS	FIML	OLS	3SLS
	50	OLS	2SLS	OLS	2SLS\3SLS
	100	OLS	FIML	2SLS	FIML
	250	2SLS	2SLS	OLS\2SLS	2SLS
	500	2SLS	2SLS	2SLS	3SLS
0.99	10	OLS	2SLS	OLS	3SLS
	20	OLS/2SLS	OLS/3SLS	2SLS	OLS
	30	OLS	2SLS	OLS	3SLS
	50	2SLS	2SLS	OLS\2SLS	2SLS
	100	2SLS	FIML	2SLS	2SLS\3SLS
	250	OLS	2SLS/FIML	OLS	2SLS
	500	2SLS	FIML	OLS	FIML

Source: Obtained from author's simulated results of bias

Comment: From Table 1, the levels of multicollinearity that exist between the exogenous variables were classified to levels with sample sizes categories to obtain Table 1a.

From Table 1a, with exact identified equation, it was observed that at all the levels of correlation between the error terms, the following are observed:
In exact identified equation, 2SLS estimator is generally best at all levels of multicollinearity with small sample size category in both distributions; except that at low multicollinearity level with large sample size in normally distributed exogenous variables at this instance 3SLS estimator is the best. Also, in high multicollinearity level with small sample size category in normally distributed exogenous variables, severe multicollinearity level with small sample size category in normally distributed exogenous variables, high multicollinearity level with medium sample size category in normally distributed exogenous variables, low multicollinearity level with small sample size category in uniformly distributed exogenous variables, high multicollinearity level with small sample size category in uniformly distributed exogenous variables, low multicollinearity level with medium sample size category in uniformly distributed exogenous variables, and severe multicollinearity level with large sample size category in uniformly distributed exogenous variables, at these instances OLS estimator is the best.

Table 1a: Table showing the multicollinearity level and sample size categories under bias criterion

Identification Status	distribution	Multicoll. Level	Sample Size: Small	Medium	Large
Exact	Normal	Low(0.3)	OLS/2SLS	2SLS	**3SLS**
		Moderate (0.6)	O/2SLS	2SLS	2/3SLS
		High (0.9)	**OLS**	**OLS**	2SLS
		Severe (0.99)	**OLS**	2SLS	O/2SLS
	Uniform	Low(0.3)	**OLS**	**OLS**	2SLS
		Moderate (0.6)	O/2SLS	O/2SLS	2SLS
		High (0.9)	**OLS**	O/2SLS	2SLS
		Severe (0.99)	O/2SLS	2SLS	**OLS**
Over	Normal	Low(0.3)	**3SLS**	2/3SLS	2SLS/FIML
		Moderate (0.6)	2SLS	**3SLS/FIML**	2/3SLS/FIML
		High(0.9)	2SLS/FIML	2SLS/FIML	2SLS
		Severe (0.99)	2/3SLS	2SLS/FIML	**FIML**
	Uniform	Low(0.3)	2SLS	2SLS	2/FIML
		Moderate (0.6)	2/FIML	2SLS	2/FIML
		High(0.9)	2/3SLS	2/3SLS/FIML	2/3SLS
		Severe (0.99)	**O/3SLS**	2SLS	2/FIML

Source: Table 1

In over identified equation 2SLS estimator is the best at all levels of multicollinearity with sample size categories of both distributions, except with low multicollinearity level with small sample size category in normally distributed exogenous variables, moderate multicollinearity level with medium sample size category in normally distributed exogenous variables, severe multicollinearity level with large sample size in uniform distribution, low multicollinearity level with large sample size in normally distributed exogenous variables and severe multicollinearity level with small sample size in uniform distribution, at these instances 3SLS, 3SLS/FIML, FIML and OLS/3SLS estimator is respectively the best.

Variance examination of the model parameters

The summary of the performances of the estimators having summed the rank of the variance over all the parameters in each equation was obtained for each selected sample size at each selected level of multicollinearity.

For each sample size, the summed ranks values were further added over the equations, and preferred estimator(s) under both exactly and over identification model with the two exogenous variables were obtained (Appendix 3) and the summary of the preferred estimators are given in Table 2.

From Table 2a, it was observed that in exact identified equation, OLS estimator is generally the best with both distributions at all levels of multicollinearity with sample size categories. In over identified equation, OLS estimator is generally the best at all levels of multicollinearity with sample size categories of both distributions, except with moderate multicollinearity level with small sample size in normally distributed exogenous variables, moderate multicollinearity level with medium sample size in normally distributed exogenous variables, severe multicollinearity level with large sample size in normally distributed exogenous variables, severe multicollinearity level with small sample size in uniformly distributed exogenous variables and high multicollinearity level with large sample size category in uniformly distributed exogenous variables, at these instances 2SLS estimator is the best.

Table 2: Summary of the preferred estimator(s) for the variance criterion when there is multicollinearity alone in th model

Multicoll. level	Sample Sizes	Normal exogenous variables: Exactly identified	Normal exogenous variables: Over-identified	Uniform exogenous variables: Exactly identified	Uniform exogenous variables: Over - identified
0.3	10	OLS	OLS	OLS	OLS
	20	OLS	OLS/2SLS	OLS	OLS/2SLS
	30	OLS	OLS/2SLS	OLS	2SLS
	50	OLS	OLS	OLS	2SLS
	100	OLS	OLS	OLS	OLS
	250	OLS	2SLS	OLS	OLS/2SLS
	500	OLS	OLS	OLS	OLS
0.6	10	OLS	2SLS	OLS	OLS
	20	OLS	OLS/2SLS	OLS	2SLS
	30	OLS	2SLS	OLS	OLS/2SLS
	50	OLS	OLS/2SLS	OLS	OLS
	100	OLS	2SLS	OLS	FIML
	250	OLS	2SLS	OLS	OLS/2SLS
	500	OLS	OLS	OLS	OLS
0.9	10	OLS	OLS/2SLS	OLS	OLS
	20	OLS	OLS	OLS	2SLS
	30	OLS	OLS	OLS	OLS
	50	OLS	OLS	OLS	OLS
	100	OLS	OLS/2SLS	OLS	OLS
	250	OLS	OLS	OLS	2SLS
	500	OLS	OLS	OLS	OLS/2SLS
0.99	10	OLS	2SLS	OLS	2SLS
	20	OLS	OLS	OLS	2SLS
	30	OLS	OLS	OLS	OLS/2SLS
	50	OLS	2SLS	OLS	2SLS
	100	OLS	OLS/2SLS	OLS	OLS
	250	OLS	2SLS	OLS	OLS
	500	OLS	2SLS	OLS	OLS

Source: Obtained from simulated results of variance

Comment: From Table 2, the levels of multicollinearity that exist between the exogenous variables was classified to levels with sample sizes categories to obtain Table 2a.

Table 2a: Table that shows the multicollinearitylevel and sample size categories under variance criterion

Identification Status	Distrib.	Multicoll. Level	Sample Size: Small	Medium	Large
Exact	Normal	Low(0.3)	OLS	OLS	OLS
		Moderate (0.6)	OLS	OLS	OLS
		High (0.9)	OLS	OLS	OLS
		Severe (0.99)	OLS	OLS	OLS
	Uniform	Low(0.3)	OLS	OLS	O/2SLS
		Moderate (0.6)	OLS	OLS	OLS
		High (0.9)	OLS	OLS	OLS
		Severe (0.99)	OLS	OLS	OLS
Over	Normal	Low(0.3)	OLS	OLS	O/2SLS
		Moderate (0.6)	**2SLS**	**2SLS**	O/2SLS
		High(0.9)	OLS	OLS	OLS
		Severe (0.99)	O/2SLS	**2SLS**	**2SLS**
	Uniform	Low(0.3)	O/2SLS	O/2SLS	OLS
		Moderate (0.6)	O/2SLS	O/FIML	OLS
		High(0.9)	O/2SLS	OLS	**2SLS**
		Severe (0.99)	**2SLS**	O/2SLS	OLS

Source: Table 2

Mean absolute error examination of the model parameters

The summary of the performances of the estimators having summed the rank of the mean absolute error over all the parameters in each equation was obtained for each selected sample size at each selected level of multicollinearity.

For each sample size, the summed ranks values were further added over the equations, and preferred estimator(s) under both exactly and over identification model with the two exogenous variables were obtained (Appendix 4) and the summary of the preferred estimators are given in Table 3.

From Table 3a, it was observed that in exact identified equation, OLS estimator is generally the best in both distributions at all levels of multicollinearity with sample size categories except with moderate multicollinearity level with medium sample size category, at this instance, 2SLS estimator is the best.

Table 3: Summary of the preferred estimator(s) for the mean absolute error criterion when there is multicollinearity alone in the model

Multicollinearity	N	Normal Exogenous Variables		Uniform Exogenous Variables	
		Exact identified	**Over identified**	**Exact identified**	**Over – identified**
0.3	10	OLS	2SLS	OLS	OLS
	20	OLS	O/2SLS	OLS	2SLS
	30	2SLS	OLS	OLS	2SLS
	50	OLS	OLS	OLS	2SLS
	100	OLS	2SLS	OLS	OLS
	250	OLS	2SLS	OLS	2SLS\3SLS\FIML
	500	OLS	OLS	OLS	2SLS
0.6	10	OLS	2SLS	2SLS	FIML
	20	OLS	OLS	OLS	2SLS
	30	OLS	FIML	OLS	2SLS
	50	OLS	FIML	2SLS	2SLS
	100	OLS	2SLS	2SLS	FIML
	250	OLS	2SLS	OLS	2SLS
	500	2SLS\3SLS	FIML	OLS	2SLS
0.9	10	OLS	OLS	OLS	OLS
	20	OLS	OLS	OLS	2SLS
	30	OLS	OLS	OLS	2SLS
	50	OLS	OLS	OLS	OLS
	100	OLS	OLS	OLS	OLS
	250	OLS	OLS	OLS	2SLS
	500	OLS	OLS	OLS	2SLS
0.99	10	OLS	OLS	OLS	2SLS
	20	OLS	OLS	OLS	2SLS
	30	OLS	2SLS	OLS	OLS
	50	OLS	2SLS	OLS	2SLS
	100	OLS	2SLS	OLS	2SLS
	250	OLS	OLS	OLS	2SLS
	500	OLS	2SLS	OLS	OLS

Source: Obtained from simulated results of mean absolute error

Comment: From Table 3, the levels of multicollinearity that exist between the exogenous variables was classified to levels with sample sizes categories to obtain Table 3a.

Table 3a: Table that shows the multicollinearity level and sample size categories under mean absolute error criterion

Identification Status	distribution	Multicoll. Level	Sample Size		
			Small	**Medium**	**Large**
Exact	Normal	Low(0.3)	O/2SLS	OLS	OLS
		Moderate (0.6)	OLS	OLS	O/2/3SLS
		High (0.9)	OLS	OLS	OLS
		Severe (0.99)	OLS	OLS	OLS
	Uniform	Low(0.3)	OLS	OLS	OLS
		Moderate (0.6)	O/2SLS	**2SLS**	OLS
		High (0.9)	OLS	OLS	OLS
		Severe (0.99)	OLS	OLS	OLS
Over	Normal	Low(0.3)	O/2SLS	O/2SLS	O/2SLS
		Moderate (0.6)	O/2SLS/FIML	2/FIML	2SLS/FIML
		High(0.9)	OLS	**OLS**	**OLS**
		Severe (0.99)	O/2SLS	2SLS	O/2SLS
	Uniform	Low(0.3)	O/2SLS	O/2SLS	2SLS
		Moderate (0.6)	2SLS/FIML	2SLS/FIML	2SLS
		High(0.9)	O/2SLS	**OLS**	2SLS
		Severe (0.99)	O/2SLS	2SLS	O/2SLS

Source: Table 3

In over identified equation, 2SLS estimator is generally the best at all levels of multicollinearity with sample size categories of both distributions, except with high multicollinearity level with medium sample size category in normally distributed exogenous variables, high multicollinearity level with large sample size category in normally distributed exogenous variables and high multicollinearity level with medium sample size category in uniformly distributed exogenous variables. at these instances OLS estimator is the best.

Mean squared error examination of the model parameters

The summary of the performances of the estimators having summed the rank of mean squared error over all the parameters in each equation was obtained for each selected sample size at each selected level of multicollinearity (Appendix 4). For each sample size, the summed ranks values were further added over the equations, and preferred estimator(s) under both exactly and over identification model with the two exogenous variables were obtained and the summary of the preferred estimators are given in Table 4.

From Table 4a, it was observed that in exact identified equation, OLS estimator is generally the best with both distributions at all levels of multicollinearity with sample size categories except with moderate multicollinearity level with medium sample size category of uniformly distributed exogenous variables, at this instance, 2SLS estimator is the best.

In over identified equation, OLS estimator is generally the best at all levels of multicollinearity and sample size categories of both distributions, except with low multicollinearity level with large sample size category and moderate multicollinearity level with large sample size category in uniformly distributed exogenous variables, at these instances 2SLS or FIML estimator is the best.

Table 4: Summary of the preferred estimator(s) for the mean squared error criterion when there is multicollinearity alone in the model

Multicollinearity	Sample sizes	Normal Exogenous Variables		Uniform Exogenous Variables	
		Exactly identified	Over – identified	Exactly identified	Over identified
	10	OLS	OLS	OLS	OLS
	20	OLS	OLS	OLS	OLS
	30	OLS	OLS	OLS	OLS
	50	OLS	OLS	OLS	OLS
0.3	100	OLS	OLS	OLS	OLS
	250	OLS	OLS	OLS	2SLS
	500	OLS	OLS	OLS	2SLS
	10	OLS	OLS	2SLS	FIML
	20	OLS	OLS	OLS	2SLS
0.6	30	OLS	2SLS	OLS	OLS
	50	OLS	2SLS	2SLS	FIML
	100	OLS	OLS	2SLS\3SLS	FIML
	250	OLS	OLS	OLS	2SLS
	500	3SLS	FIML	OLS	OLS
	10	OLS	OLS	OLS	OLS
	20	OLS	OLS	OLS	OLS
	30	OLS	OLS	OLS	OLS
0.9	50	OLS	OLS	OLS	OLS
	100	OLS	OLS	OLS	OLS
	250	OLS	OLS	OLS	OLS
	500	OLS	OLS	OLS	OLS
	10	OLS	OLS	OLS	OLS
	20	OLS	OLS	OLS	OLS
	30	OLS	OLS	OLS	OLS
0.99	50	OLS	OLS	OLS	2SLS
	100	OLS	OLS	OLS	3SLS
	250	OLS	OLS	OLS	OLS
	500	OLS	OLS	OLS	OLS

Source: Obtained from simulated results of MSE

Comment: From Table 4, the levels of multicollinearity that exist between the exogenous variables was classified to levels with sample sizes categories to obtain Table 4a.

Table 4a: Table that shows the multicollinearity level and sample size categories under mean square error criterion

Identification Status	distribution	Multicoll. Level	Sample Size		
			Small	**Medium**	**Large**
Exact	Normal	Low(0.3)	OLS	OLS	OLS
		Moderate (0.6)	OLS	OLS	O/2SLS
		High (0.9)	OLS	OLS	OLS
		Severe (0.99)	OLS	OLS	OLS
	Uniform	Low(0.3)	OLS	OLS	OLS
		Moderate (0.6)	O/2SLS	**2SLS**	OLS
		High (0.9)	OLS	OLS	OLS
		Severe (0.99)	OLS	OLS	OLS
Over	Normal	Low(0.3)	OLS	OLS	OLS
		Moderate (0.6)	O/2SLS	O/2SLS	OLS/FIML
		High(0.9)	OLS	OLS	OLS
		Severe (0.99)	OLS	OLS	OLS
	Uniform	Low(0.3)	OLS	OLS	**2SLS**
		Moderate (0.6)	O/2SLS/FIML	**FIML**	O/2SLS
		High(0.9)	OLS	OLS	OLS
		Severe (0.99)	OLS	2/3SLS	OLS

Source: Table 4

Under bias criterion with exact and over identified equation in both distribution, 2SLS estimator is generally the best, except that the performances of estimator was affected by the presence of multicollinearity at some sample sizes category and multicollinearity levels, at these instances , 3SLS or 3SLS/FIML or FIML or OLS/3SLS estimator is the best.

Under variance criterion with exact and over identified equation in both distribution, OLS estimator is generally the best, only the performances of estimation techniques in uniformly distributed exogenous variables was affected by the presence of multicollinearity at some sample sizes category and multicollinearity levels, at these instance , 2SLS estimator is the best.

Under mean absolute error criterion with exact and over identified equation of both distribution, OLS estimator is generally the best, the performances of estimation techniques in equations with either uniformly or normally distributed exogenous variables was affected by the presence of multicollinearity at some sample sizes category and multicollinearity levels, at these instance , 2SLS estimator is the best.

Under mean square error criterion, in exact identified equation, OLS estimator is generally the best in both distribution but at medium sample size in uniformly distributed exogenous variables, the performances of estimator was affected by the presence of multicollinearity, at this instance, 2SLS is the best. But in over identified equation, OLS estimator is generally the best in both distribution but at medium and large sample size in uniformly distributed exogenous variables, the performances of estimation techniques was affected by the presence of multicollinearity, at these instance , FIML or 2SLS estimator is the best.

Conclusion

The performances of the estimators with normally distributed exogenous variables are slightly different from the performances of the estimator with uniformly distributed exogenous variables over multicollinearity levels and sample size categories. This implies that, presence of multicollinearity in the equations of the model and the distribution of exogenous variables in simultaneous equation of the model with identification status, affect the performances of the estimation techniques of the model. Hence identification status, distribution of exogenous variables of the equation in the model and existence of multicollinearity between the exogenous variables must also be taken into consideration before the choice of estimators to be used in estimating the parameters under simultaneous equation model.

Conflict of Interest

Authors declare that there are no conflicts of interest

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APPENDIXES

Appendix 1: Performances of the estimators under the bias criterion when there is presence of multicollinearity alone in the model

ρ	n	Estimator	Normal Exogenous Variables						Uniform Exogenous Variables					
			Exactly identified			Over – identified			Exactly identified			Over - identified		
			Eq1	Eq3	Total	Eq2	Eq4	Total	Eq1	Eq3	Total	Eq2	Eq4	Total
0.3	10	OLS	5	8	13	9	12	21	4	6	**10**	9	10	19
		2SLS	7	4	**11**	6	9	15	8	6	14	6	4	**10**
		3SLS	14	12	26	3	6	**9**	12	12	24	3	9	12
		FIML	14	16	30	12	3	15	16	16	32	12	7	19
	20	OLS	5	5	**10**	9	12	21	6	6	**12**	9	11	20
		2SLS	7	7	14	4	8	**12**	6	6	**12**	4	4	**8**
		3SLS	16	14	30	5	7	**12**	12	14	26	5	7	12
		FIML	12	14	26	12	3	15	16	14	30	12	8	20
	30	OLS	7	4	**11**	12	12	24	4	6	**10**	12	12	24
		2SLS	5	8	13	9	6	15	8	6	14	3	5	**8**
		3SLS	13	15	28	6	3	**9**	12	12	24	6	8	14
		FIML	15	13	28	3	9	12	16	16	32	9	5	14
	50	OLS	8	8	16	12	12	24	5	6	**11**	11	12	23
		2SLS	4	4	**8**	9	6	15	7	6	13	4	3	**7**
		3SLS	12	13	25	6	3	**9**	14	15	29	7	8	15
		FIML	16	15	31	3	9	12	14	13	27	8	7	15
	100	OLS	16	9	25	12	12	24	6	5	**11**	9	12	21
		2SLS	4	4	**8**	3	3	**6**	6	7	13	3	9	**12**
		3SLS	8	14	22	6	6	12	14	15	29	6	6	**12**
		FIML	12	13	25	9	9	18	14	13	27	12	3	15
	250	OLS	16	16	32	12	12	24	7	8	15	12	12	24
		2SLS	5	12	17	4	3	**7**	5	4	**9**	6	4	**10**
		3SLS	7	4	**11**	5	6	11	15	13	28	9	9	18
		FIML	10	10	20	7.5	7.5	15	10	10	20	7.5	7.5	15
		OLS	16	16	32	12	12	24	7	8	15	12	12	24
	500	2SLS	4	12	16	7	5	12	5	4	**9**	9	3	12
		3SLS	8	5	**13**	8	9	17	13	14	27	6	9	15
	500	FIML	12	7	19	3	4	**7**	15	14	29	3	6	**9**
0.6	10	OLS	4	4	**8**	9	11	20	5	6	**11**	9	11	20
		2SLS	8	8	16	3	5	**8**	7	6	13	4	7	**11**
		3SLS	12	14	26	6	8	14	12	12	24	5	6	**11**
		FIML	16	14	30	12	6	18	16	16	32	12	6	18
	20	OLS	8	5	13	9	10	19	5	6	**11**	9	10	19
		2SLS	4	7	**11**	3	3	**6**	7	6	13	3	6	**9**
		3SLS	12	16	28	6	6	12	12	12	24	6	7	13
		FIML	16	12	28	12	11	23	16	16	32	12	7	19
	30	OLS	4	6	**10**	9	12	21	8	5	13	12	12	24
		2SLS	8	6	14	3	9	**12**	4	7	**11**	8	9	17
		3SLS	15	14	29	6	6	**12**	12	13	25	7	5	12
		FIML	13	14	27	12	3	15	16	15	31	3	4	**7**
	50	OLS	8	8	16	12	12	24	5	6	**11**	9	10	19
		2SLS	4	4	**8**	3	7	10	7	6	13	3	3	**6**
		3SLS	12	13	25	9	8	17	12	13	25	6	9	15
		FIML	16	15	31	6	3	**9**	16	15	31	12	8	20
	100	OLS	7	10	17	12	12	24	5	8	13	9	12	21
		2SLS	5	6	**11**	9	3	12	7	4	**11**	3	3	**6**
		3SLS	13	12	25	5	6	**11**	14	12	26	6	9	15
		FIML	15	12	27	4	9	13	14	16	30	12	6	18
	250	OLS	16	16	32	12	12	24	5	8	13	12	12	24
		2SLS	5	4	**9**	7	3	**10**	7	4	**11**	9	3	12
		3SLS	7	10	17	4	6	**10**	12	13	25	6	9	15
		FIML	12	10	22	7	9	16	16	15	31	3	6	**9**
	500	OLS	16	16	32	12	12	24	7	8	15	12	12	24
		2SLS	4	12	16	7	6	13	5	4	**9**	3	5	**8**
		3SLS	8	4	**12**	8	9	17	13	14	27	7	8	15
		FIML	12	8	20	3	3	**6**	15	14	29	8	5	13
0.9	10	OLS	4	4	**8**	9	10	19	6	6	**12**	10	6	16
		2SLS	8	8	16	6	3	**9**	6	6	**12**	7	3	**10**
		3SLS	14	13	27	3	9	12	12	13	25	10	10	20
		FIML	14	15	29	12	8	20	16	15	31	3	11	14
	20	OLS	7	4	**11**	11	12	23	4	4	**8**	5	7	**12**
		2SLS	5	8	13	7	8	15	8	8	16	6	6	**12**
		3SLS	12	12	24	6	5	**11**	12	12	24	7	5	**12**
		FIML	16	16	32	6	5	**11**	16	16	32	12	12	24
		OLS	4	5	**9**	12	12	24	4	4	**8**	9	8	17
		2SLS	8	7	15	9	3	12	8	8	16	6	4	10

0.9	**30**	3SLS	12	13	25	6	8	14	12	14	26	3	6	**9**
		FIML	16	15	31	3	7	**10**	16	14	30	12	12	24
	50	OLS	4	4	**8**	9	12	21	4	4	**8**	9	11	20
		2SLS	8	8	16	4	3	**7**	8	8	16	6	3	**9**
		3SLS	12	14	26	5	6	11	12	14	26	3	6	**9**
		FIML	16	14	30	12	9	21	16	14	30	12	10	22
	100	OLS	5	4	**9**	12	12	24	7	6	13	12	12	24
		2SLS	7	8	15	9	3	12	5	6	**11**	7	5	12
		3SLS	12	14	26	6	9	15	12	12	24	6	8	14
		FIML	16	14	30	3	6	**9**	16	16	32	5	5	**10**
	250	OLS	7	8	15	12	12	24	7	5	**12**	12	12	24
		2SLS	5	4	**9**	6	5	**11**	5	7	**12**	3	3	**6**
		3SLS	16	14	30	9	4	13	12	14	26	9	8	17
		FIML	12	14	26	3	9	12	16	14	30	6	7	13
	500	OLS	8	8	16	12	12	24	6	8	14	12	12	24
		2SLS	4	4	**8**	3	5	**8**	6	4	**10**	6	9	15
		3SLS	13	13	26	9	8	17	13	14	27	3	4	**7**
		FIML	15	15	30	6	5	11	15	14	29	9	5	14
0.99	**10**	OLS	5	4	**9**	6	8	14	4	4	**8**	5	9	14
		2SLS	7	8	15	3	5	**8**	8	8	16	7	6	13
		3SLS	12	16	28	9	5	14	12	12	24	6	6	**12**
		FIML	16	12	28	12	12	24	16	16	32	12	9	21
	20	OLS	6	6	**12**	3	9	**12**	7	7	14	5	8	**13**
		2SLS	6	6	**12**	9	6	15	5	5	**10**	6	9	15
		3SLS	12	12	24	6	6	**12**	12	12	24	7	7	14
		FIML	16	16	32	12	9	21	16	16	32	12	6	18
	30	OLS	4	5	**9**	10	9	19	5	7	**12**	8	9	17
		2SLS	8	7	15	7	4	**11**	7	6	13	9	6	15
		3SLS	12	13	25	4	8	12	13	14	27	7	6	**13**
		FIML	16	15	31	9	9	18	15	13	28	6	9	15
	50	OLS	7	9	16	8	10	18	7	5	**12**	8	11	19
		2SLS	5	9	**14**	5	3	**8**	5	7	**12**	3	6	**9**
		3SLS	14	9	23	11	9	20	13	13	26	7	8	15
		FIML	14	13	27	6	8	14	15	15	30	12	5	17
	100	OLS	7	6	13	12	12	24	6	8	14	9	9	18
		2SLS	5	6	**11**	9	3	12	6	4	**10**	4	8	**12**
		3SLS	12	13	25	6	8	14	14	15	29	5	7	**12**
		FIML	16	15	31	3	7	**10**	14	13	27	12	6	18
	250	OLS	6	4	**10**	12	10	22	4	4	**8**	7	10	17
		2SLS	6	8	14	9	3	**12**	8	8	16	7	3	**10**
		3SLS	15	12	27	6	8	14	12	13	25	4	9	13
		FIML	13	16	29	3	9	**12**	16	15	31	12	8	20
	500	OLS	6	8	14	12	10	22	4	6	**10**	12	9	21
		2SLS	6	4	**10**	9	3	12	8	6	14	8	8	16
		3SLS	13	14	27	6	9	15	12	13	25	7	7	14
		FIML	15	14	29	3	8	**11**	16	15	31	3	6	**9**

Source: Computed from author's simulated results of Bias

Appendix 2: Performances of the estimators under the variance criterion when there is presence of multicollinearity alone in the model

ρ	n	Estimator	Normal Exogenous Variables						Uniform Exogenous Variables					
			Exactly identified			Over - identified			Exactly identified			Over - identified		
			Eq1	Eq3	Total	Eq2	Eq4	Total	Eq1	Eq3	Total	Eq2	Eq4	Total
0.3	10	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	7	**10**
		2SLS	8	8	16	6	8	14	8	8	16	6	6	12
		3SLS	15	13	28	9	7	16	12	12	24	9	8	17
		FIML	13	15	28	12	6	18	16	16	32	12	9	21
	20	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	9	**12**
		2SLS	8	8	16	6	6	**12**	8	8	16	6	6	**12**
		3SLS	12	12	24	9	9	18	13	15	28	9	5	14
		FIML	16	16	32	12	6	18	15	13	28	12	10	22
	30	OLS	4	4	**8**	3	7	**10**	4	4	**8**	3	9	**12**
		2SLS	8	8	16	6	4	**10**	8	8	16	6	8	14
		3SLS	16	14	30	9	9	18	15	13	28	9	7	16
		FIML	12	14	26	12	10	22	13	15	28	12	6	18
	50	OLS	4	4	**8**	3	6	**9**	4	4	**8**	3	9	12
		2SLS	8	8	16	6	5	11	8	8	16	6	4	**10**
		3SLS	12	15	27	9	7	16	12	14	26	9	8	17
		FIML	16	13	29	12	12	24	16	14	30	12	9	21
		OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	7	**10**
		2SLS	8	8	16	6	7	13	8	8	16	6	5	11

	100	3SLS	12	12	24	9	8	17	12	12	24	9	9	18
		FIML	16	16	32	12	6	18	16	16	32	12	9	21
	250	OLS	4	4	**8**	3	8	11	4	4	**8**	3	9	**12**
		2SLS	8	8	16	6	4	**10**	8	8	16	6	6	**12**
		3SLS	12	15	27	9	7	16	12	12	24	9	5	14
		FIML	16	13	29	12	11	23	16	16	32	12	10	22
	500	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	9	**12**
		2SLS	8	8	16	6	7	13	8	8	16	6	7	13
		3SLS	12	12	24	9	6	15	15	12	27	9	8	17
		FIML	16	16	32	12	8	20	13	16	29	12	6	18
0.6	10	OLS	4	4	**8**	3	9	12	4	4	**8**	3	9	**12**
		2SLS	8	8	16	6	4	**10**	9	8	17	12	4	16
		3SLS	12	13	25	9	7	16	11	14	25	9	7	16
		FIML	16	15	31	12	10	22	16	14	30	6	10	16
	20	OLS	4	4	**8**	3	7	**10**	4	4	**8**	3	9	12
		2SLS	9	8	17	6	4	**10**	8	8	16	6	5	**11**
		3SLS	13	13	26	9	9	18	15	15	30	9	6	15
		FIML	14	15	29	12	10	22	13	13	26	12	10	22
	30	OLS	4	4	**8**	3	9	12	4	4	**8**	3	7	**10**
		2SLS	8	8	16	6	4	**10**	8	8	16	6	4	**10**
		3SLS	12	13	25	10	7	17	15	14	29	9	9	18
		FIML	16	15	31	11	10	21	13	14	27	12	10	22
	50	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	9	**12**
		2SLS	8	8	16	8	4	**12**	8	10	18	12	5	17
		3SLS	13	15	28	7	7	14	12	13.5	25.5	9	9.5	18.5
		FIML	15	13	28	12	10	22	16	12.5	28.5	6	6.5	12.5
	100	OLS	4	4	**8**	3	9	12	4	4	**8**	3	9	12
		2SLS	8	8	16	6	4	**10**	8	14	22	12	6	18
		3SLS	12	14	26	9	9	18	12	10.5	22.5	9	10	19
		FIML	16	14	30	12	8	20	16	11.5	27.5	6	5	**11**
	250	OLS	4	4	**8**	3	9	12	4	4	**8**	3	9	**12**
		2SLS	8	8	16	6	4	**10**	8	8	16	6	6	**12**
		3SLS	12	13	25	9	9	18	14	13	27	10	9	19
		FIML	16	15	31	12	8	20	14	15	29	11	6	17
	500	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	7	**10**
		2SLS	8	16	24	11	4	15	8	8	16	6	4	**10**
		3SLS	12	10	22	10	8	18	12	14	26	9	9	18
		FIML	16	10	26	6	9	15	16	14	30	12	10	22
0.9	10	OLS	4	4	**8**	3	7	**10**	4	4	**8**	3	5	**8**
		2SLS	8	8	16	6	4	**10**	8	8	16	6	8	14
		3SLS	12	12	24	9	9	18	12	12	24	9	8	17
		FIML	16	16	32	12	10	22	16	16	32	12	9	21
	20	OLS	4	4	**8**	3	8	**11**	4	4	**8**	3	9	12
		2SLS	8	8	16	6	6	12	8	8	16	6	5	**11**
		3SLS	12	12	24	9	7	16	12	12	24	9	8	17
		FIML	16	16	32	12	9	21	16	16	32	12	8	20
	30	OLS	4	4	**8**	3	5	**8**	4	4	**8**	3	9	**12**
		2SLS	8	8	16	6	4	**10**	8	8	16	6	8	14
		3SLS	12	13	25	9	10	19	12	14	26	9	6	15
		FIML	16	15	31	12	11	23	16	14	30	12	7	19
	50	OLS	4	4	**8**	3	7	**10**	4	4	**8**	3	6	**9**
		2SLS	8	8	16	6	6	12	8	8	16	6	4	10
		3SLS	12	12	24	9	8	17	12	14	26	9	8	17
		FIML	16	16	32	12	9	21	16	14	30	12	12	24
	100	OLS	4	4	**8**	3	8	**11**	4	4	**8**	3	7	**10**
		2SLS	8	8	16	7	4	**11**	8	8	16	6	6	12
		3SLS	12	15	27	8	7	15	12	13	25	9	8	17
		FIML	16	13	29	12	11	23	16	15	31	12	9	21
	250	OLS	4	4	**8**	3	7	**10**	4	4	**8**	3	9	12
		2SLS	8	8	16	6	6	12	8	8	16	6	4	**10**
		3SLS	12	12	24	9	7	16	14	13	27	9	8	17
		FIML	16	16	32	12	10	22	14	15	29	12	9	21
	500	OLS	4	4	**8**	3	5	**8**	4	4	**8**	3	7	**10**
		2SLS	8	8	16	6	6	12	8	8	16	6	4	**10**
		3SLS	12	12	24	9	7	16	12	13	25	9	10	19
		FIML	16	16	32	12	12	24	16	15	31	12	9	21
0.99	10	OLS	4	4	**8**	3	8	11	4	4	**8**	3	9	12
		2SLS	8	8	16	6	4	**10**	8	8	16	6	4	**10**
		3SLS	12	13	25	9	7	16	13	13	26	9	10	19
		FIML	16	15	31	12	11	23	15	15	30	12	7	19
	20	OLS	4	4	**8**	3	8	**11**	4	4	**8**	3	9	12
		2SLS	8	8	16	6	7	13	8	8	16	6	4	**10**
		3SLS	12	13	25	9	8	17	12	13.5	25.5	10	8	18
		FIML	16	15	31	12	7	19	16	14.5	30.5	11	9	20

	30	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	7	**10**
		2SLS	8	8	16	6	8	14	8	8	16	6	4	**10**
		3SLS	14	15	29	9	8	17	13	14	27	9	9	18
		FIML	14	13	27	12	5	17	15	14	29	12	10	22
	50	OLS	4	4	**8**	3	9	12	4	4	**8**	3	9	12
		2SLS	8	8	16	6	4	**10**	8	10	18	6	4	**10**
		3SLS	14	14	28	9	10	19	12	10	22	9	9	18
		FIML	14	14	28	12	7	19	16	16	32	12	8	20
	100	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	9	**12**
		2SLS	8	8	16	7	5	**12**	8	8	16	9	6	15
		3SLS	12	12	24	8	8	16	13	15	28	6	8	14
		FIML	16	16	32	12	8	20	15	13	28	12	7	19
	250	OLS	4	4	**8**	3	6	**9**	4	4	**8**	3	9	**12**
		2SLS	8	8	16	6	7	13	8	8	16	6	8	14
		3SLS	15	14	29	9	8	17	12	13	25	9	6	15
		FIML	13	14	27	12	9	21	16	15	31	12	7	19
	500	OLS	4	4	**8**	3	9	12	4	4	**8**	3	6	**9**
		2SLS	8	8	16	6	5	**11**	8	8	16	6	7	13
		3SLS	14	15	29	9	9	18	12	15	27	9	8	17
		FIML	14	13	27	12	7	19	16	13	29	12	9	21

Source: Computed from author's simulated results of Variance

Appendix 3: Performances of the estimators under the mean absolute error criterion when there is presence of multicollinearity alone in the model

ρ	n	Estimator	Normal Exogenous Variables						Uniform Exogenous Variables					
			Exactly identified			Over - identified			Exactly identified			Over - identified		
			Eq1	Eq3	Total	Eq2	Eq4	Total	Eq1	Eq3	Total	Eq2	Eq4	Total
0.3	10	OLS	4	4	8	8	3	11	4	4	8	3	3	6
		2SLS	8	8	16	3	6	9	8	8	16	6	6	12
		3SLS	15	12	27	7	9	16	12	12	24	9	9	18
		FIML	13	16	29	12	12	24	16	16	32	12	12	24
	20	OLS	4	4	8	6	3	9	4	4	8	12	12	24
		2SLS	8	8	16	3	6	9	8	8	16	3	3	6
		3SLS	12	12	24	9	9	18	13	15	28	7	6	13
		FIML	16	16	32	12	12	24	15	13	28	8	9	17
	30	OLS	4	4	8	3	3	6	4	4	8	9	12	21
		2SLS	8	8	16	6	6	12	8	8	16	3	3	6
		3SLS	16	13	29	9	9	18	15	13	28	6	6	12
		FIML	12	15	27	12	12	24	13	15	28	12	9	21
	50	OLS	4	4	8	3	3	6	4	4	8	9	9	18
		2SLS	8	8	16	6	6	12	8	8	16	3	4	7
		3SLS	12	15	27	9	9	18	12	14	26	6	7	13
		FIML	16	13	29	12	12	24	16	14	30	12	10	22
	100	OLS	4	4	8	9	12	21	4	4	8	3	3	6
		2SLS	8	8	16	3	3	6	8	8	16	6	6	12
		3SLS	12	12	24	6	6	12	12	12	24	9	9	18
		FIML	16	16	32	12	9	21	16	16	32	12	12	24
	250	OLS	4	4	8	6	8	14	4	4	8	12	12	24
		2SLS	8	8	16	3	3	6	8	8	16	9	3	12
		3SLS	12	13	25	9	7	16	12	12	24	6	6	12
		FIML	16	15	31	12	12	24	16	16	32	3	9	12
	500	OLS	4	4	8	3	5	8	4	4	8	12	12	24
		2SLS	8	8	16	6	4	10	8	8	16	3	3	6
		3SLS	12	12	24	9	9	18	12	12	24	9	6	15
		FIML	16	16	32	12	12	24	16	16	32	6	9	15
0.6	10	OLS	4	4	8	9	10	19	16	12	28	12	12	24
		2SLS	8	8	16	3	3	6	4	4	8	9	3	12
		3SLS	12	14	26	6	7	13	8	12	20	6	7.5	13.5
		FIML	16	14	30	12	10	22	12	12	24	3	7.5	10.5
	20	OLS	4	4	8	3	5	8	6	4	10	12	12	24
		2SLS	8	8	16	6	4	10	6	8	14	3	3	6
		3SLS	12	12	24	9	9	18	15	14	29	6	6	12
		FIML	16	16	32	12	12	24	13	14	27	9	9	18
	30	OLS	16	7	23	12	12	24	4	4	8	9	5	14
		2SLS	4	5	9	9	3	12	8	8	16	3	4	7
		3SLS	8	13	21	6	7	13	15	12	27	6	9	15
		FIML	12	15	27	3	8	11	13	16	29	12	12	24
	50	OLS	6	4	10	12	12	24	16	16	32	12	12	24
		2SLS	6	8	14	9	3	12	4	6	10	9	3.5	12.5
		3SLS	13	14	27	6	9	15	8	8.5	16.5	6	8	14
		FIML	15	14	29	3	6	9	12	9.5	21.5	3	6.5	9.5
		OLS	4	4	8	12	11	23	16	16	32	12	12	24

	100	2SLS	8	8	16	3	3	6	4	8	12	9	4.5	13.5
		3SLS	12	13	25	6	8	14	8	7.5	15.5	6	8.5	14.5
		FIML	16	15	31	9	8	17	12	8.5	20.5	3	5	8
	250	OLS	4	4	8	9	9	18	6	5	11	12	12	24
		2SLS	8	8	16	4	4	8	6	7	13	3	3	6
		3SLS	12	13	25	5	8	13	13	13	26	7	7	14
		FIML	16	15	31	12	9	21	15	15	30	8	8	16
	500	OLS	16	16	32	12	12	24	4	4	8	12	5	17
		2SLS	4	10	14	9	3	12	8	8	16	3	4	7
		3SLS	8	6	14	6	9	15	12	14	26	6	11	17
		FIML	12	8	20	3	6	9	16	14	30	9	10	19
0.9	10	OLS	4	4	8	3	3	6	4	4	8	3	3	6
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	12	12	24	9	9	18	12	12	24	9	9	18
		FIML	16	16	32	12	12	24	16	16	32	12	12	24
	20	OLS	4	4	8	3	3	6	4	4	8	6	9	15
		2SLS	8	8	16	6	6	12	8	8	16	3	4	7
		3SLS	12	12	24	9	9	18	12	12	24	9	7	16
		FIML	16	16	32	12	12	24	16	16	32	12	10	22
	30	OLS	4	4	8	3	3	6	4	4	8	6	5	11
		2SLS	8	8	16	6	6	12	8	8	16	3	4	7
		3SLS	12	12	24	9	9	18	12	13	25	9	9	18
		FIML	16	16	32	12	12	24	16	15	31	12	12	24
	50	OLS	4	4	8	3	3	6	4	4	8	3	3	6
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	12	12	24	9	9	18	12	14	26	9	9	18
		FIML	16	16	32	12	12	24	16	14	30	12	12	24
	100	OLS	4	4	8	3	5	8	4	4	8	3	3	6
		2SLS	8	8	16	6	4	10	8	8	16	6	6	12
		3SLS	12	13	25	9	9	18	12	12	24	9	9	18
		FIML	16	15	31	12	12	24	16	16	32	12	12	24
	250	OLS	4	4	8	3	5	8	4	4	8	9	7	16
		2SLS	8	8	16	6	4	10	8	8	16	3	4	7
		3SLS	12	12	24	9	9	18	14	13	27	6	9	15
		FIML	16	16	32	12	12	24	14	15	29	12	10	22
	500	OLS	4	4	8	3	3	6	4	4	8	9	5	14
		2SLS	8	8	16	6	6	12	8	8	16	3	4	7
		3SLS	12	12	24	9	9	18	12	12	24	6	10	16
		FIML	16	16	32	12	12	24	16	16	32	12	11	23
0.99	10	OLS	4	4	8	3	5	8	4	4	8	12	12	24
		2SLS	8	8	16	6	4	10	8	8	16	3	3	6
		3SLS	12	13	25	9	9	18	12	13	25	6	8	14
		FIML	16	15	31	12	12	24	16	15	31	9	7	16
	20	OLS	4	4	8	3	3	6	4	4	8	8	9	17
		2SLS	8	8	16	6	6	12	8	8	16	3	4	7
		3SLS	12	14	26	9	9	18	12	12	24	7	8	15
		FIML	16	14	30	12	12	24	16	16	32	12	9	21
	30	OLS	4	4	8	9	10	19	4	4	8	3	3	6
		2SLS	8	8	16	3	3	6	8	8	16	6	6	12
		3SLS	14	15	29	6	9	15	12	13	25	9	9	18
		FIML	14	13	27	12	8	20	16	15	31	12	12	24
	50	OLS	4	4	8	9	9	18	6	4	10	12	12	24
		2SLS	8	8	16	3	4	7	6	8	14	3	3	6
		3SLS	13	14	27	6	9	15	13	13	26	6	8	14
		FIML	15	14	29	12	8	20	15	15	30	9	7	16
	100	OLS	4	4	8	7	9	16	4	4	8	11	12	23
		2SLS	8	8	16	6	4	10	8	8	16	6	3	9
		3SLS	12	12	24	5	7	12	12	14	26	3	8	11
		FIML	16	16	32	12	10	22	16	14	30	10	7	17
	250	OLS	4	4	8	3	3	6	4	4	8	12	7	19
		2SLS	8	8	16	6	6	12	8	8	16	3	3	6
		3SLS	14	14	28	9	9	18	12	13	25	6	8	14
		FIML	14	14	28	12	12	24	16	15	31	9	12	21
	500	OLS	4	4	8	12	12	24	4	4	8	3	5	8
		2SLS	8	8	16	3	3	6	8	8	16	6	4	10
		3SLS	13	12	25	6	8	14	12	14	26	9	9	18
		FIML	15	16	31	9	7	16	16	14	30	12	12	24

Source: Computed from author's simulated results of Absolute mean error

Appendix 4: Performances of the estimators under the mean squared error criterion when there is presence of multicollinearity alone in the model

ρ	n	Estimator	Normal Exogenous Variables						Uniform Exogenous Variables					
			Exactly identified			Over - identified			Exactly identified			Over - identified		
			Eq1	Eq3	Total	Eq2	Eq4	Total	Eq1	Eq3	Total	Eq2	Eq4	Total
0.3	10	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	15	12	27	9	9	18	12	12	24	9	9	18
		FIML	13	16	29	12	12	24	16	16	32	12	12	24
	20	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	5	**8**
		2SLS	8	8	16	6	6	12	8	8	16	6	4	10
		3SLS	13	12	25	9	9	18	13	15	28	9	9	18
		FIML	15	16	31	12	12	24	15	13	28	12	12	24
	30	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	16	13	29	9	9	18	14	13	27	9	9	18
		FIML	12	15	27	12	12	24	14	15	29	12	12	24
	50	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	12	14	26	9	9	18	13	14	27	9	9	18
		FIML	16	14	30	12	12	24	15	14	29	12	12	24
	100	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	12	12	24	9	9	18	12	12	24	9	9	18
		FIML	16	16	32	12	12	24	16	16	32	12	12	24
	250	OLS	4	4	**8**	3	3	**6**	4	4	**8**	11	12	23
		2SLS	8	8	16	6	6	12	8	8	16	3	3	**6**
		3SLS	12	15	27	9	9	18	12	12	24	6	6	12
		FIML	16	13	29	12	12	24	16	16	32	10	9	19
	500	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	10	13
		2SLS	8	8	16	6	6	12	8	8	16	6	3	**9**
		3SLS	12	12	24	9	9	18	15	12	27	9	7	16
		FIML	16	16	32	12	12	24	13	16	29	12	10	22
0.6	10	OLS	4	4	**8**	3	3	**6**	16	4	20	12	12	24
		2SLS	8	8	16	6	6	12	5	8	**13**	9	3	12
		3SLS	12	13	25	9	9	18	7	14	21	6	7	13
		FIML	16	15	31	12	12	24	12	14	26	3	8	**11**
	20	OLS	4	4	**8**	3	3	**6**	4	4	**8**	12	12	24
		2SLS	8	8	16	6	6	12	8	8	16	3	3	**6**
		3SLS	12	13	25	9	10	19	15	15	30	6	6	12
		FIML	16	15	31	12	11	23	13	13	26	9	9	18
	30	OLS	8	4	**12**	12	12	24	4	4	**8**	3	3	**6**
		2SLS	6	8	14	3	3	**6**	8	8	16	6	6	12
		3SLS	11	13	24	7	6	13	15	14	29	9	10	19
		FIML	15	15	30	8	9	17	13	14	27	12	11	23
	50	OLS	4	4	**8**	12	3	15	16	16	32	12	12	24
		2SLS	8	8	16	5	6	**11**	4	6	**10**	9	3	12
		3SLS	13	15	28	4	10	14	8	8.5	16.5	6	8	14
		FIML	15	13	28	9	11	20	12	9.5	21.5	3	7	**10**
	100	OLS	4	6	**10**	3	3	**6**	16	16	32	12	12	24
		2SLS	8	6	14	6	6	12	4	10	**14**	9	3	12
		3SLS	12	14.5	26.5	9	11	20	8	6	**14**	6	9	15
		FIML	16	13.5	29.5	12	10	22	12	8	20	3	6	**9**
	250	OLS	4	4	**8**	3	3	**6**	4	4	**8**	12	12	24
		2SLS	8	8	16	6	6	12	8	8	16	3	3	**6**
		3SLS	12	13	25	9	11	20	14	13	27	7	9	16
		FIML	16	15	31	12	10	22	14	15	29	8	6	14
	500	OLS	16	16	32	12	12	24	4	4	**8**	3	3	**6**
		2SLS	4	12	16	8	3	11	8	8	16	6	6	12
		3SLS	8	5.5	**13.5**	7	8	15	12	14	26	9	9	18
		FIML	12	6.5	18.5	3	7	**10**	16	14	30	12	12	24
	10	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	12	12	24	9	10	19	12	12	24	9	9	18
		FIML	16	16	32	12	11	23	16	16	32	12	12	24
	20	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	12	12	24	9	9	18	12	12	24	9	10	19
		FIML	16	16	32	12	12	24	16	16	32	12	11	23
	30	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	12	13	25	9	9	18	12	14	26	9	9	18

0.9		FIML	16	15	31	12	12	24	16	14	30	12	12	24
	50	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	12	12	24	9	9	18	12	14	26	9	9	18
		FIML	16	16	32	12	12	24	16	14	30	12	12	24
	100	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	7	6	13	8	8	16	6	6	12
		3SLS	12	15	27	8	9	17	12	13	25	9	9	18
		FIML	16	13	29	12	12	24	16	15	31	12	12	24
	250	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	12	12	24	9	9	18	14	13	27	9	10	19
		FIML	16	16	32	12	12	24	14	15	29	12	11	23
	500	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	12	12	24	9	9	18	12	13	25	9	11	20
		FIML	16	16	32	12	12	24	16	15	31	12	10	22
0.99	10	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	4	**7**
		2SLS	8	8	16	6	6	12	8	8	16	6	5	11
		3SLS	12	13	25	9	9	18	13	13	26	9	11	20
		FIML	16	15	31	12	12	24	15	15	30	12	10	22
	20	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	12	13	25	9	9	18	12	13.5	25.5	10	10	20
		FIML	16	15	31	12	12	24	16	14.5	30.5	11	11	22
	30	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	14	15	29	9	11	20	12	13	25	9	9	18
		FIML	14	13	27	12	10	22	16	15	31	12	12	24
	50	OLS	4	4	**8**	3	3	**6**	4	4	**8**	12	10	22
		2SLS	8	8	16	6	6	12	8	10	18	3	3	**6**
		3SLS	14	14	28	9	11	20	12	10	22	6	7	13
		FIML	14	14	28	12	10	22	16	16	32	9	10	19
	100	OLS	4	4	**8**	3	3	**6**	4	4	**8**	6	12	18
		2SLS	8	8	16	7	6	13	8	8	16	9	3	12
		3SLS	12	12	24	8	9	17	13	15	28	3	6	**9**
		FIML	16	16	32	12	12	24	15	13	28	12	9	21
	250	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	6	12	8	8	16	6	6	12
		3SLS	15	15	30	9	9	18	12	13	25	9	9	18
		FIML	13	13	26	12	12	24	16	15	31	12	12	24
	500	OLS	4	4	**8**	3	5	**8**	4	4	**8**	3	3	**6**
		2SLS	8	8	16	6	4	10	8	8	16	6	6	12
		3SLS	14	14	28	9	10	19	12	15	27	9	10	19
		FIML	14	14	28	12	11	23	16	13	29	12	11	23

Source: Computed from author's simulated results of mean square error