EFFECTS OF MAGNETIC FIELD ON NATURAL CONVECTIVE STEADY FLOW PAST AN IRREGULAR VERTICAL CHANNEL IN THE PRESENCE OF VISCOUS DISSIPATION

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Abstract: Magnetic field effects on Steady free Convective flow through an irregular vertical channel in the presence of viscous dissipation is considered. The dimensional governing equations of the problem involving continuity, momentum and energy equations were non-dimensionalized. The dimensionless non-linear coupled differential equations were solved using Adomian Decomposition Method (ADM) with mathematica 11.0 software. On velocity and temperature, the effects of different fluid parameters were observed and presented both graphically and in tabular forms. It was observed that increase in viscous dissipation parameter enhanced both velocity and temperature profiles while a rise in magnetic parameter retarded both velocity and temperature distributions of the fluid.

Keywords: Constant magnetic field, ADM, natural convection, irregular channel

Introduction

Magnetic field has gained the attentions of wide range of researchers due to its applications in the field of mechanical and chemical engineering. Ira *et al*. (2018) examined mass and heat transfer effects on unsteady natural convective flow through an inclined surface in the presence of heat generation. Their results revealed that magnetic field slowdown the flow field. Magnetic field effects on free convective heat transfer past Anisotropic river beds was considered by Yovogan *et al*. (2018). It was observed that magnetic field decelerated heat transfer, temperature and velocity profiles. Saidulu (2018) investigated numerical solutions of magnetic field and Hall current effects on free convective past an inclined semi-infinite vertical plate. Finite difference technique was used to obtain the solutions. The results showed that both temperature and velocity increase with a hike in viscous dissipation parameter. Shah *et al*. (2018) studied magnetic field effect on viscous fluid flow past a vertically moving plate in the presence of double convection and constant temperature. Moses *et al*. (2017) discussed magnetic field and Brinkman number effects on laminar convection through vertical channel plate. Magnetic field effect on convective viscous dusty fluid flow with chemical reaction was studied by Krishna *et al*. (2013). It was reported that increase in magnetic field decelerated velocity profiles.

The Significance of dissipation function in fluid flow has attracted the attention of numerous researchers due to its applications in engineering in controlling cooling rate. For instance, Zigta (2018) presented viscous dissipation and chemical reaction effects on MHD fluid flow in the presence of thermal radiation. Their results revealed that concentration distribution is retarded by a rise in viscous dissipation parameter. Emeka (2019) examined impact of Soret and viscous dissipation on MHD fluid flow in the presence of thermal radiation. Reddy *et al*. (2016) investigated viscous dissipation and thermal radiation effects on convective MHD Micropolar fluid flow through a moving plate with injection/suction. They reported that thermal radiation decreases the rate of heat transfer. Agunbiade and Dada (2019) considered impacts of dissipation on rotatory Rivlin-Ericksen fluid flow through a permeable vertical plate in the presence of chemical reaction. The result showed that increase in Eckert number and rotation parameter enhanced resultant velocity profiles. Ajibade and Kabir (2019) addressed effect of viscous dissipation on convective steady couette heat generating fluid flow through a vertical channel.

Free or natural convection is a kind of mechanism where fluid motion is not as a result of external source but is due to density differences within the fluid as a result of temperature gradients. Free convection has gained global interest of researchers because of its evident in nature, as well as, engineering applications; for instance, in the ecological process. One of its applications is free air cooling that does not involve fans. Therefore, Okuyade and Okor (2018) examined Natural convective MHD fluid flow through a vertically heated non-permeable surface in the presence of double-diffusion and thermal radiation. It was reported that a rise in Prandtl decreased the fluid temperature profiles. Umavathi *et al*. (2018) studied natural convection of micropolar of an electrically-conducting fluid flow between parallel permeable vertical plates. Differential transform was used to obtain the results. Shamshuddin and Satya (2018) considered secondary and Primary flows on unsteady free convective MHD rotating Micropolar fluid flow via an inclined plate. They reported that thermal boundary layer thickness increased with a hike in Eckert number. Dada and Agunbiade (2016) investigated impacts of thermal radiation on natural convective Rivlin-Ericksen fluid flow through a permeable vertical plate in the presence of chemical reaction. The results showed that temperature and velocity distributions is slowed down with a rise in thermal radiation parameter. Sharma (2017) presented effects of chemical reaction on natural convection MHD flow via a vertical porous channel with influence of heat source and thermal radiation. Gbadeyan *et al*. (2017) discussed effects of thermal radiation on natural convective mass and heat transfer of chemically reacting fluid flow via an irregular channel. The results shown that increase in chemical reaction and thermal radiation slowed down velocity profiles.

Therefore, considering the gaps left uncovered in the above reviewed work of the earlier researchers, the objective of this study is to examine effects of constant Magnetic field on steady free Convective flow through an irregular vertical channel in the presence of viscous dissipation. The governing equations of the flow is highly coupled and non-linear differential equations, hence Adomian decomposition method is applied.

Mathematical Analysis

We studied an incompressible,laminar electrically conducting, natural convective steady fluid flow through an irregular vertical channel with viscous dissipation. The channel is considered to be infinite, in x-axis is vertically upward oriented, the fluid is bounded by irregular wall ($\eta^* = \varepsilon^* \cos\omega x$) and flat wall ($\eta^* = \delta$) with distance δ apart, while η-axis is perpendicular to x-axis. B_o, the uniform magnetic field, is normal to η-axis. The wavelength of the wall roughness is given as $\omega = \frac{2\pi}{k}$, where the wave coefficient is k. Insignificance of induced magnetic field when compared to applied magnetic field is as a result of the assumption that magnetic Reynolds number is very small. In comparison with other chemical species, diffusing species is very low in magnitude; hence, effects of thermal-diffusion and Diffusion-thermo are neglected. The flow configuration for this problem is given below:

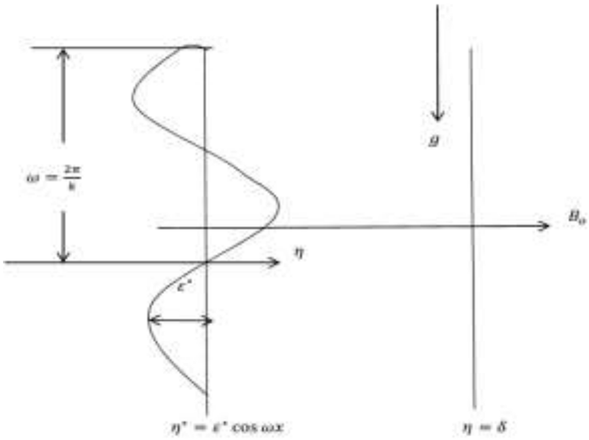

Fig. 1: Geometry of the problem

Based on Boussinesq approximation, dimensional governing equations of the flow consisting of momentum and energy equations are:

$$\nu\frac{\partial^2\xi^*}{\partial\eta^{*2}} + g\beta_T(T^* - T_o) - \frac{\sigma B_o^2\xi^*}{\rho} = 0 \quad (1)$$

$$\frac{\alpha}{\rho C_P}\frac{\partial^2 T^*}{\partial\eta^{*2}} + \frac{\mu}{\rho C_P}\left(\frac{\partial\xi^*}{\partial\eta^*}\right)^2 - \frac{Q_o}{\rho C_P}(T^* - T_o) = 0 \quad (2)$$

The associated boundary conditions are:

$$\left.\begin{array}{llll} \xi^* = U_o, & T^* = T_\delta & at & \eta^* = \varepsilon^*\cos\omega x, \\ \xi^* = 0, & T^* = T_o & at & \eta^* = \delta \end{array}\right\} \quad (3)$$

The following non-dimensional variables are introduced to transform the governing equations to dimensionless form.

$$\left.\begin{array}{l} \eta = \frac{\eta^*}{\delta}, \xi = \frac{\xi^*}{U_o}, \vartheta = \frac{T^* - T_o}{T_\delta - T_o}, \phi = \frac{Q_o\delta^2}{\alpha}, G_r = \frac{g\beta\delta^2(T_\delta - T_o)}{\nu U_o} \\ E_c = \frac{U_o}{C_P(T_\delta - T_o)}, P_r = \frac{\mu C_P}{\alpha}, M = \frac{\sigma B_o^2\delta^2}{\rho\nu}, \varepsilon = \frac{\varepsilon^*}{\delta} \end{array}\right\} \quad (4)$$

Using equation (4), the governing equations becomes:

$$\frac{d^2\xi}{d\eta^2} + G_r\vartheta - M\xi = 0 \quad (5)$$

$$\frac{d^2\vartheta}{d\eta^2} + E_c P_r\left(\frac{d\xi}{d\eta}\right)^2 - \phi\vartheta = 0 \quad (6)$$

with the boundary conditions;

$$\left.\begin{array}{llll} \xi = 1, & \vartheta = 1 & at & \eta = \varepsilon\cos\omega x, \\ \xi = 0, & \vartheta = 0 & at & \eta = 1 \end{array}\right\} \quad (7)$$

Method of solution

This problem involves coupled, non-linear ordinary differential equations, hence; Adomian Decomposition method (ADM) is used because of it flexibility in handling both linear and non-linear differential equations (Chen and Lu (2004)). The solution is obtained by taken $\varepsilon = 0$ which is the limit for a flat smooth wall. Therefore, equation (5) becomes

$$L^{-1}(L\xi) = L^{-1}(M\xi - G_r\vartheta) \quad (8)$$

where $L = \frac{d^2}{d\eta^2}$

and

$$L^{-1} = \int_0^\eta\int_0^s d\eta ds$$

Hence, equation (8) gives

$$\xi(\eta) = \xi(0) + \xi'(0)\eta + \int_0^\eta\int_0^s (M\xi - G_r\vartheta)d\eta ds \quad (9)$$

Applying the boundary conditions (7) to equation (9) leads to

$$\xi(\eta) = 1 + b_1\eta + \int_0^\eta\int_0^s (M\xi - G_r\vartheta)d\eta ds \quad (10)$$

where $b_1 = \xi'(0)$

From equation (10), the initial approximation and the recurrence relation is given as:

$$\xi_0 = 1 + b_1\eta \quad (11)$$

$$\xi_{n+1} = \int_0^\eta\int_0^s (M\xi_n - G_r\vartheta_n)d\eta ds; n = 0,1,2,\ldots \quad (12)$$

Equation (6) can be written as

$$\vartheta'' = \phi\vartheta - E_c P_r(\xi')^2 \quad (13)$$

Hence

$$\vartheta(\eta) = \vartheta(0) + \vartheta'(0)\eta + \int_0^\eta\int_0^s (\phi\vartheta - E_c P_r(\xi')^2)d\eta ds \quad (14)$$

Using the boundary conditions (7), equation (14) becomes

$$\vartheta(\eta) = 1 + b_2\eta + \int_0^\eta\int_0^s (\phi\vartheta - E_c P_r(\xi')^2)d\eta ds \quad (15)$$

where $b_2 = \vartheta'(0)$

Similarly, the initial approximation and recurrence relation are:

$$\vartheta_0 = 1 + b_2\eta \quad (16)$$

$$\vartheta_{n+1} = \int_0^\eta\int_0^s (\phi\vartheta_n - E_c P_r(\xi_n')^2)d\eta ds; n = 0,1,2,\ldots \quad (17)$$

The approximate solutions for equations (5) and (6), which converges at $n = 5$, can be written in series forms as:

$$\xi = \sum_{n=0}^5 \xi_n \quad (18)$$

$$\vartheta = \sum_{n=0}^5 \vartheta_n \quad (19)$$

Results and Discussion

The ADM with the aid of MATHEMATICA 11.0 software is used to obtain solutions to the ordinary differential equations (5) and (6) with the boundary conditions (7). The effects of different parameters involved are examined for velocity and temperature profiles and are presented both in tabular and graphical forms. Parameters considered are: magnetic parameter M, Prandtl number P_r, dimensionless heat absorption coefficient ϕ, Grashof number for heat transfer G_r and dissipation parameter E_c. The following values are used for the computations: $G_r = 8.0$, $M = 0.2$, $P_r = 0.71$, $\phi = 2$, and $E_c = 0.6$.

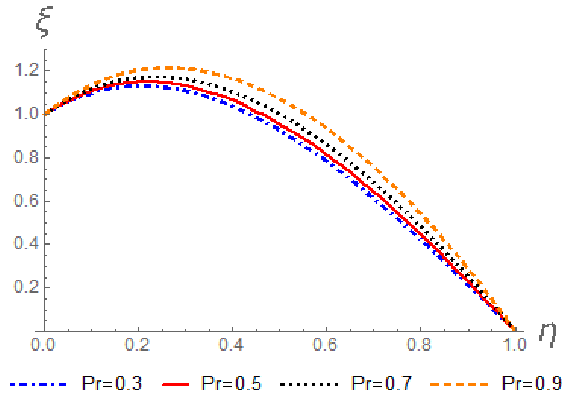

Fig. 2: Velocity profiles for different values of *Pr*

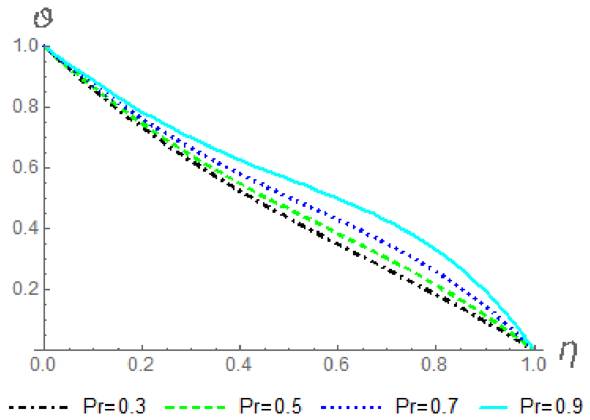

Fig. 3: Temperature profiles for different values of *Pr*

Figures 2 and 3 depict the effect of Prandtl number P_r on velocity and temperature profiles. It is obvious from these figures that a rise in Prandtl number P_r enhanced both velocity and temperature profiles. Physically, increase in Prandtl number makes the thermal conductivity of the fluid to decrease. Thus, at higher Prandtl number, the rate at which diffusion of heat takes place at heated surface is more rapid.

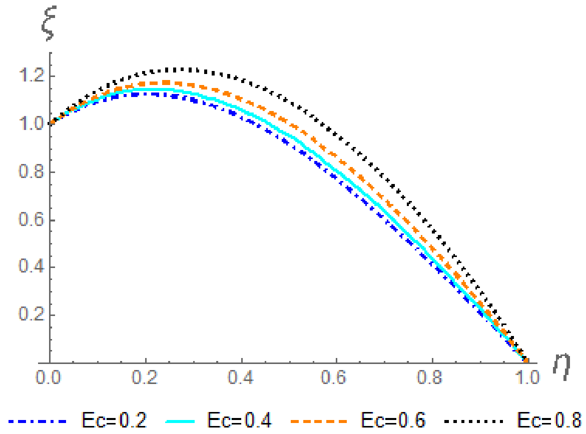

Fig. 4: Velocity profiles for different values of *Ec*

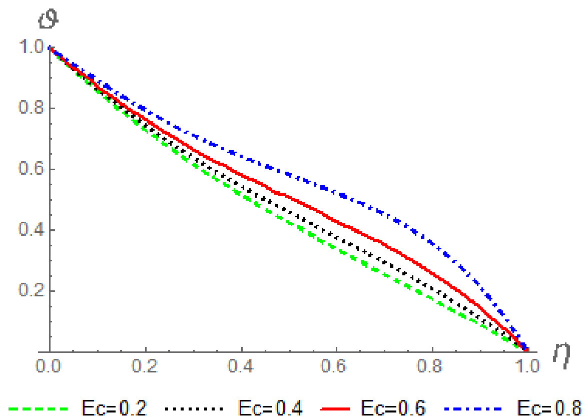

Fig. 5: Temperature profiles for different values of *Ec*

The effect of viscous dissipation on velocity and temperature distributions is exhibited in Figs. 4 and 5. Eckert number E_c shows the relationship that exist between kinetic energy and enthalpy. It is noteworthy that increase in viscous dissipation accelerate velocity and temperature profiles. The effect of dissipation on the fluid makes energy to increase and consequently, increase the fluid temperature and buoyancy force. Hence, from these Figures, velocity and temperature increase with higher values of E_c

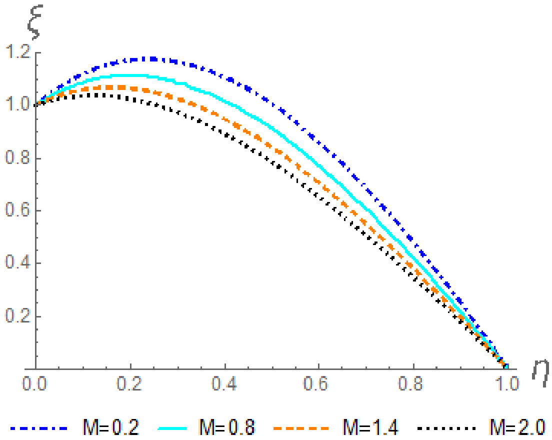

Fig. 6: Velocity profiles for different values of *M*

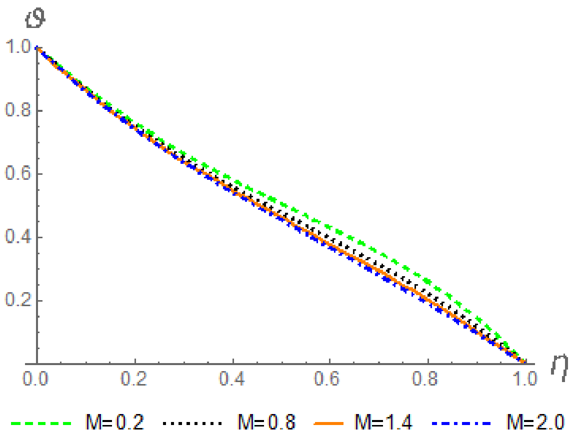

Fig. 7: Temperature profiles for different values of *M*

Figures 6 and 7 reveal the influence of magnetic parameter M on velocity and temperature profiles. It can be seen from Fig. 6 that velocity is slowdown with increase in magnetic parameter. A drag force, popularly known as Lorentz force is associated to electrically conducting fluid with application of magnetic field. Therefore, the velocity of the fluid experienced retardation due to this drag force that opposed the fluid transport. Likewise, temperature decelerated with increase in M.

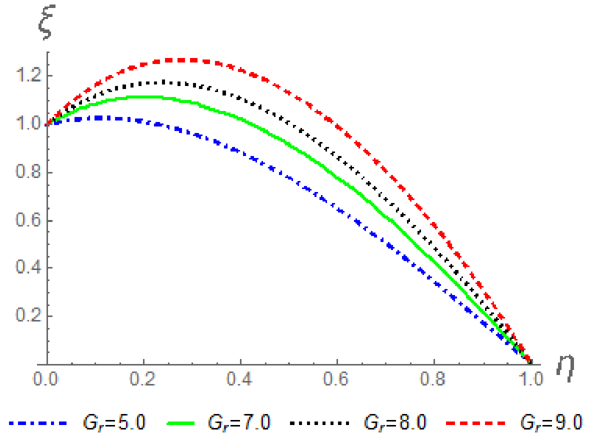

Fig. 8: Velocity profiles for different values of Gr

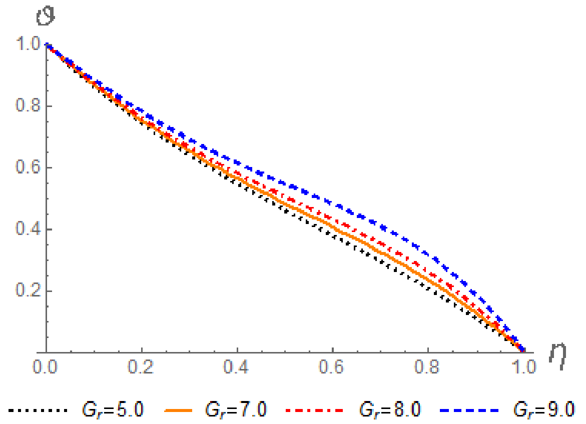

Fig. 9: Temperature profiles for different values of Gr

Effect of Grashof number for heat transfer Gr is presented in Figures 8 and 9. It is observed from these Figures that a rise in Gr speedup both velocity and temperature profiles. Physically, higher values of Grashof number enhances buoyancy force which consequently, within the channel, increases hydrodynamics. It is acknowledged that, positive values of Gr correspond to the cooling.

Figures 10 and 11 show the variation of heat absorption coefficient ϕ on velocity and temperature distributions. Finally, from these Figures, it is detected that temperature and velocity decelerated with increase in ϕ. It is noticed that temperature profile is decreased by buoyancy force when heat is absorbed.

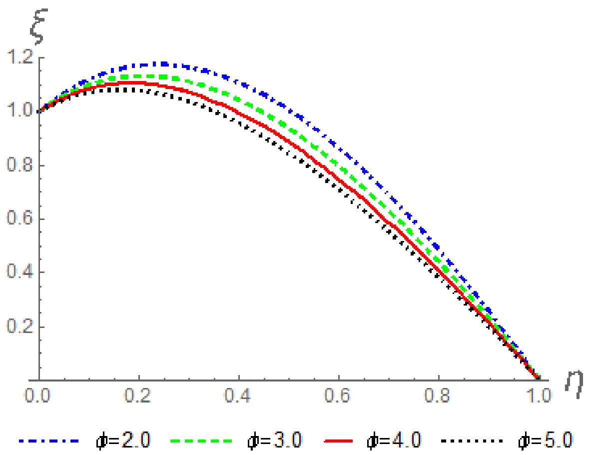

Fig. 10: Velocity profiles for different values of ϕ

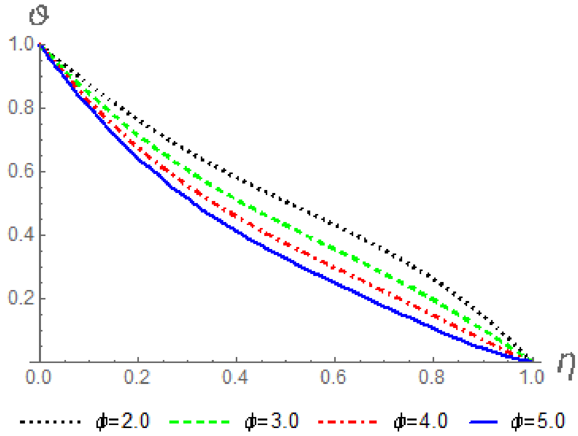

Fig. 11: Temperature profiles for different values of ϕ

The variation of different fluid parameters (ϕ, G_r, P_r, E_c and M) on Skin-friction and Nusselt number is presented in Table 1. At $\eta = 0$, it is revealed that an increase in G_r, P_r, and E_c enhanced both skin friction and heat transfer coefficient. While, skin friction and Nusselt number is retarded by increasing ϕ and M.

Table 1: Values of Skin-friction and Nusselt number for different parameters

ϕ	G_r	P_r	E_c	M	S_f	Nu
2	8	0.71	0.6	0.2	1.56873	-1.31595
3	8	0.71	0.6	0.2	1.35812	-1.66004
5	8	0.71	0.6	0.2	1.06533	-2.20209
2	5	0.71	0.6	0.2	0.49438	-1.45029
2	7	0.71	0.6	0.2	1.17876	-1.38564
2	9	0.71	0.6	0.2	2.05823	-1.17650
2	8	0.3	0.6	0.2	1.36413	-1.50138
2	8	0.5	0.6	0.2	1.44772	-1.42401
2	8	0.7	0.6	0.2	1.56182	-1.32205
2	8	0.71	0.2	0.2	1.34194	-1.52258
2	8	0.71	0.4	0.2	1.43517	-1.43543
2	8	0.71	0.6	0.2	1.56873	-1.31595
2	8	0.71	0.6	0.2	1.56873	-1.31595
2	8	0.71	0.6	0.8	1.20321	-1.39725
2	8	0.71	0.6	1.4	0.90548	-1.43918

Table 2: Numerical comparison of the work of Jha and Ajibade (2010), Ajibade and kabir (2019) and the present work
($P_r = 0.71$, $G_r = 8.0$ and $y = \eta = 0.5$)

	Jha and Ajibade		Ajibade and Kabir		Present work	
	Temp.	**Velocity**	**Temp.**	**Velocity**	**Temp.**	**Velocity**
-1.0	0.569747	1.067976	0.568331	1.064149	0.569747	1.057980
-0.5	0.532965	1.029436	0.535019	1.029058	0.532965	1.027440
0.5	0.470299	0.975218	0.470377	0.975282	0.470299	0.975218
1.0	0.443409	0.952725	0.443485	0.953983	0.443409	0.952724

The comparison of the present work with the work of Jha and Ajibade (2010) and Ajibade and kabir (2019) is shown in Table 2. It is observed from Table 2 that the present work is in agreement with the work of Jha and Ajibade (2010) and Ajibade and kabir (2019) by setting viscous dissipation and magnetic field parameter to zero.

Conclusion
An analysis is carried out on the Magnetic field effect on Natural Convective, incompressible and electrically conducting fluid flow past an irregular vertical channel in the presence of viscous dissipation. The solutions to the dimensionless governing equations were obtained using ADM with MATHEMATICA 11.0 software. The effects of various parameters on fluid properties were examined and illustrated graphically, as well as, in tabular form. The results shown that velocity, temperature, Skin friction and Nusselt number were speedup with increase in viscous dissipation parameter while they were retarded with increase in Magnetic parameter. Viscous dissipation and magnetic field are relevant, particularly in problems involving free Convective through an irregular channel which is not unpopular in nature.

Nomenclature
U_o is the velocity of the wavy wall (ms^{-1})
C_p specific heat at constant pressure ($Jkg^{-1} K^{-1}$)
B_0 magnetic induction (tesla)
T^* dimensional temperature (K)
T_δ^* channel wall dimensional temperature (K)
T_o temperature at the left wall (K)
Q_o dimensional heat absorption/generation coefficient ($Kgm^{-1}s^{-3}K^{-1}$)
g gravitational acceleration (m/s^2)
G_r Grashof number for heat transfer
E_c Eckert number
P_r Prandtl number
M magnetic parameter
Greek Symbols
ξ^* dimensional velocity components in the η^* directions (ms^{-1})
ξ non-dimensional velocity components in the η directions
η^* dimensional distance perpendicular to the plate (m)
η non-dimensional distance perpendicular to the plate
ϑ non-dimensional fluid temperature
δ distance of the plate (m)
ρ the fluid density (kgm^{-3})
ν^* the kinematic viscosity ($m^2 s^{-1}$)
α thermal diffusivity (s^2/m^2)
β_T thermal expansion coefficient(K^{-1})
ϕ non-dimensional heat absorption parameter
μ viscosity coefficient

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