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MODELLING AND FORECASTING NIGERIAN MACRO-ECONOMIC VARIABLES WITH MULTIPLE TIME SERIES MODEL

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Abstract This study was used to determine the most suitable model for analysing inter and linear relationships between some macroeconomic variables as the nature of the model do have implications on the forecasted values and accuracy. Then, this study was used to compare the forecast and forecast evaluations of Vector autoregressive (VAR) and Multiple regression analysis models using some Nigerian macroeconomic variables. From the results, the Augmented Dickey-Fuller test showed all series were stationary at the first differencing, I(1) and only inflation rate was stationary at the ordinary level, I(0). Both models revealed the existence of inter and linear relationships but VAR models were better based on the values of the coefficient of determination. The out-sample forecast for both models indicated that Government revenue and Government expenditure exhibited a continuous rise while Inflation rate, Exchange rate and price of crude oil fluctuate on a yearly basis. The forecast evaluations results based on Root mean square forecast error (RMSE), Mean absolute error (MAE) and Mean absolute percentage error (MAPE) showed that the out-sample forecast for Vector autoregressive models was better and this indicated that nature of the model is important when analysing the relationship between macroeconomic variables.

Keywords Vector Autoregressive, Multiple regression analysis, Forecasting, Forecast evaluation, Macroeconomic variables.

Introduction

The stability of a national economy depends on the level of growth and stability of the country's macroeconomic variables (Ulvedal, 2013). The stability of the world economy has been affected by the global economic meltdown, political instability, corruption and nature (International Monetary fund IMF, 2016). The most affected are third world countries with a developing economy and Nigeria inclusive. The Nigerian economy is classified as a developing economy since government revenue is mostly generated from crude oil exportation. The global price of crude oil is unstable; the instability of government revenue affects all other key macroeconomic indicators. In the long-run, the fluctuations in the government revenue play a key role in the fluctuation of other macroeconomic indicators and in-turn, other indicators affect the government revenue as well (Okunnu *et al.*, 2017).

To have an insight into the dynamic relationship between some Nigerian macroeconomic variables, the most appropriate model to be used is the vector autoregressive model (Sims, 1980). The novelty of this model made it the appropriate model used for discussing the dynamic relationship between macroeconomic variables (Al-Sharkas, 2004; Jaafar and Ismail, 2009; Blanchard and Fischer, 1990; Adam and Mak, 1981; Ashley *et al.*, 1980).

Several pieces of research have used vector autoregressive model to investigate the dynamic relationship between some Nigerian macroeconomic variables (Adebiyi, 2005: Onwukwe and Nwafor, 2014; Lebari, 2018; Osamwonyi and Evbayiro-Osagie, 2012; Taiwo and Olatayo, 2013; Worlu and Omodero, 2017, Javed *et al.*, 2018; Elem *et al.*, 2019; Monday and Abdulkadir, 2020). The in-sample forecast and forecast performance based on forecast evaluation metric like root mean square error has been used as well to determine that vector autoregressive model is the better for measuring the dynamic relationship. But little has been done in-term of obtaining the out-sample forecast for each variable considered as the dependent variable for others as regressors and vice versa.

Having discussed this, vector autoregressive model will be used to examine the dynamic relationship between some Nigerian macroeconomic series (Government Revenue, Government Expenditure, Inflation Rate, Exchange Rate and Crude oil price) and as well obtain the out-sample forecast for each variable considered for other variables considered has regressors simultaneously. The performance of these models will be compared with the performance of Classical multiple regression based on some forecast evaluation metrics.

Materials and Methods

Vector autoregressive (VAR) model

A process y_t is said to be a vector autoregressive process of order P denoted by VAR (P) if it satisfies the equation

$$y_t = v + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \quad (1)$$

where $y_t = (y_{1t}, \ldots, y_{kt})'$ is a $(k \times 1)$ random vector, A_i are fixed $(k \times k)$ coefficient matrices, $V = (V_1, \ldots V_k)'$ is a fixed $(k \times 1)$ vector of intercept terms allowing for the possibility of a non-zero mean, $E(y_t)$.

Data exploration
The pattern and general behaviour of the series will be examined using a time plot. Test for stationarity will be carried out using augmented Dickey-Fuller methods. As well, a cointegration test will be carried out using the Johansen co-integration test.

Model diagnostics
To check for the adequacy of the estimated vector Autoregressive models obtained using the maximum likelihood estimation method, the fitted models will be subjected to model diagnostic. Vector Autoregressive lag order selection criteria will be used to choose the appropriate model order using the Akaike information criteria (AIC) and Schwarz Bayesian criteria (SBIC) given as

$$AIC(k) = ln|\hat{\Sigma}_u(k)| + \frac{2km^2}{n} \quad (2)$$

$$BIC(k) = ln|\hat{\Sigma}_u(k)| + \frac{\ln(n)\, m^2}{n} \quad (3)$$

The error term is expected to be independently distributed. This will be checked by testing the hypothesis of white noise residuals using a general-purpose portmanteau test called Q-statistic and denoted by

$$Q_h = T \sum_{k=1}^{h} tr(\hat{\gamma}_k^{l} \hat{\gamma}_0^{-1} \hat{\gamma}_k \hat{\gamma}_0^{-1}) \quad (4)$$

where $\hat{\gamma}_k = (\hat{\rho}_{ij}(k))$ is the estimated (residual) autocorrelations, $\hat{\gamma}_0$ is the contemporaneous correlations of the residuals.
While for the Classical multiple regression, R^2 is defined as

$$R^2 = 1 - \frac{SSE}{SST} \quad (5)$$

where SSE is the sum of square of error and SST is the sum of square of total. This will be used to check the adequacy of the models. Durbin-Watson statistic defined as

$$DW = \frac{\sum_{t=2}^{T}(\varepsilon_t - \varepsilon_{t-1})^2}{\sum_{t=1}^{T} \varepsilon_t^2} \quad (6)$$

where T is the number of observations will be used to measure the serial correlation in the residual.

Forecasting and forecast evaluation
Using a minimum mean square error (MMSE) method and given a VAR(1) process given as

$$y_t = A_1 y_{t-1} + u_t \quad (7)$$

The one-step-ahead forecast (Eigner, 2009) is

$$\hat{y}_N(1) = \hat{A}_1 y_N \quad (8)$$

Two step-ahead forecast is

$$\hat{y}_N(2) = \hat{A}_1\, \hat{y}_N(1) = \hat{A}_1^2\, y_N \quad (9)$$

and the h-step ahead forecast is

$$\hat{y}_N(h) = \hat{A}_1^h\, y_N \quad (10)$$

The forecast evaluation metrics used in this study are mean absolute error (MAE) defined as

$$MAE = \frac{1}{h+1} \sum_{t=s}^{h+s} (\hat{y}_t - y_t)^2 \quad (11)$$

root mean square forecast error (RMSE) defined as

$$RMSE = \sqrt{\frac{1}{h+1} \sum_{t=s}^{h+s} (\hat{y}_t - y_t)^2} \quad (12)$$

and the mean absolute percentage error is defined as

$$MAPE = \frac{100}{h+s} \sum_{t=s}^{h+s} \left| \frac{\hat{y}_t - y_t}{\hat{y}_t} \right| \quad (13)$$

where $t = s, 1+s, \ldots, h+s$. The actual and predicted values for corresponding t values are denoted by $\hat{y}_t$ *and* y_t respectively.

Results and Discussion

The time plot of Nigerian macroeconomic series obtained from Central Bank of Nigeria, Annual Statistical Bulletin from 1984 to 2019 is given in figure 1. This showed the presence of nonstationarity. It is observed that all the macroeconomic variables exhibited a long-term continuous movement in the same direction while only inflation rates have a cyclical movement and a long-term oscillation about the trend every three years. Table 1 summarizes the result of the Augmented Dickey-Fuller test and this indicated that the macroeconomic variables were stationary at the first difference and only inflation rate was stationary at the level I(0).

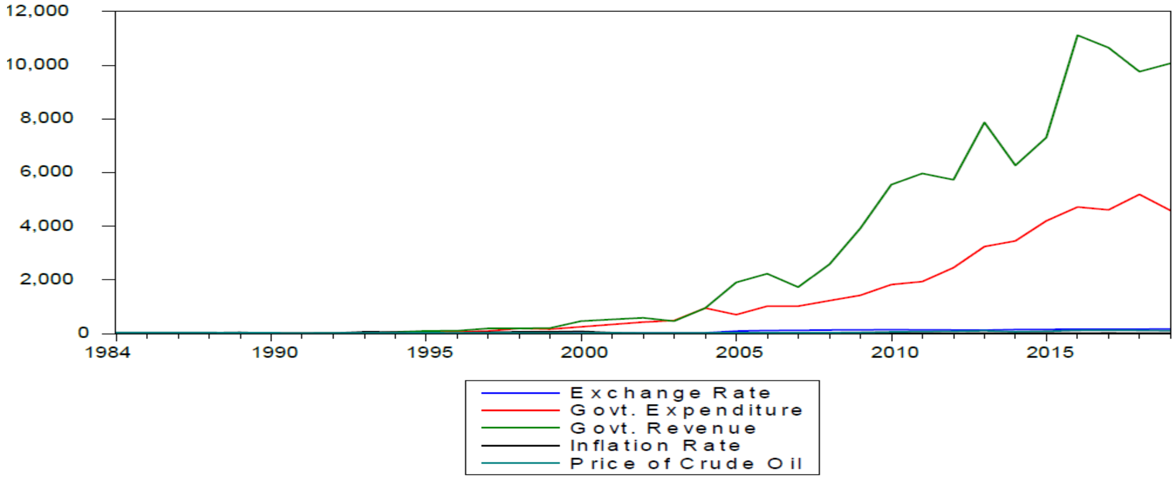

Figure 1. Time plot of some Nigerian macroeconomic variables from 1984 to 2019.

The five vector autoregressive models obtained were given in Equations 14 – 18 respectively. The value of the coefficient of determination for each model showed that more than 90% of the variation in the dependent variable was jointly explained by the independent variables. This indicated that all five models have a good fit and can be used to obtain the out-sample forecast for Nigerian macroeconomic variables.

***Table1*. Unit root test using augmented dickey-fuller (ADF)**

Economic Series	ADF Test Statistic	At 95% CL	Order of Integration
REVENUE	-9.3274	-2.9434	I(1)
GOVEXP	-6.6289	-2.9434	I(1)
GOVREV	-4.7931	-2.9434	I(1)
INF	-4.5341	-2.9434	I(0)
PRI	-4.6752	-2.9434	I(1)

Table 2 showed Lag length of order (1) is appropriate based on the result of vector autoregressive lag order selection criteria. AIC and SBIC criteria functions were both minima at the lag length of order (1), hence this makes the lag length of order (1) appropriate. Table 3 is used to present the portmanteau test result and this revealed that the residual passed white noise test since no autocorrelation is left at order (2). Based on the vector autoregressive residual normality test result in Table 4, the residuals were jointly normal since the joint skewness, kurtosis and Jarque-Bera are significant and this implies that residuals are multivariate normal.

Table 2. **Vector autoregressive lag order selection criteria**

Lag	AIC	SC
0	58.22107	58.44781
1	52.89617	54.25663*
2	52.85098*	55.34516

Table 3. **Residual autocorrelation (Portmanteau test)**

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	11.16317	NA*	11.51201	NA*	NA*
2	32.20086	NA*	33.90698	NA*	NA*
3	48.29778	0.3802	51.61359	0.2637	46
4	75.75137	0.3279	82.85388	0.1588	71
5	97.44981	0.4395	108.4270	0.1818	96
6	112.8237	0.6896	127.2173	0.3315	121

Table 4. **Vector autoregressive residual normality test**

Component	Skewness	Chi-sq	df	Prob.
1	1.434496	11.31779	1	0.0008
2	-1.053243	6.101263	1	0.0135
3	-0.878771	4.247310	1	0.0393
4	0.310225	0.529319	1	0.4669
5	-0.170195	0.159315	1	0.6898
Joint		22.35500	5	0.0004
Component	**Kurtosis**	**Chi-sq**	**df**	**Prob.**
1	7.254311	24.88634	1	0.0000
2	6.852542	20.40786	1	0.0000
3	4.089676	1.632667	1	0.2013
4	3.707558	0.688378	1	0.4067
5	2.646353	0.171966	1	0.6784
Joint		47.78722	5	0.0000
Component	**Jarque-Bera**	**Df**	**Prob.**	
1	36.20413	2	0.0000	
2	26.50912	2	0.0000	
3	5.879976	2	0.0529	
4	1.217697	2	0.5440	
5	0.331281	2	0.8474	
Joint	70.14221	10	0.0000	

$$\mathrm{EXCH}_t = \frac{0.937164}{[9.08397]}\mathrm{EXCH}_{t-1} + \frac{-0.149127}{[-1.18624]}\mathrm{INF}_{t-1} + \frac{0.012317}{[1.94463]}\mathrm{GOVEXP}_{t-1} + \frac{0.374013}{[-1.16799]}\mathrm{PRI}_{t-1} + \varepsilon_t \quad (14)$$

with $R^2 = 0.969788$

$$GOVEXP_t = \frac{0.879855}{[6.44139]} GOVEXP_{t-1} + \frac{3.765008}{[\ 1.69225]} EXCH_{t-1} + \varepsilon_t \qquad (15)$$

with $R^2 = 0.979619$

$$GOVREV_t = \frac{17.18354}{[2.76054]} EXCH_{t-1} + \frac{1.352004}{[3.53776]} GOVEXP_{t-1} \qquad (16)$$

with $R^2 = 0.966886$

$$INF_t = \frac{0.458055}{[2.69322]} INF_{t-1} + \frac{1.142304}{[2.32848]} PRI_{t-1} + \frac{2.001629}{[3.24188]} GOVREV_{t-1} + \varepsilon_t \qquad (17)$$

with $R^2 = 0.948764$

$$PRI_t = \frac{0.685283}{[2.53530]} PRI_{t-1} + \frac{0.125399}{[1.44000]} EXCH_{t-1} + \frac{0.007808}{[1.46036]} GOVEXP_{t-1} + \varepsilon_t \qquad (18)$$

with $R^2 = 0.913968$

Note: Values in [ ] are the t-statistic obtain from the vector autoregressive estimate and the other is the coefficients estimated.

The multiple regression analysis models obtained were given in given in Equations 19 –23 respectively. The value of the coefficient of determination for each model showed that more than 80% of the variation in the dependent variable was jointly explained by the independent variables. The values of Dublin Watson Statistics showed no evidence of serial correlation in the residual as the values of DW statistic were less than 2. This indicated that all the five models in Equations 19 – 23 respectively have a good fit and can be used to obtain the out-sample forecast for Nigerian macroeconomic variables.

$$EXCHRATE = 654142333579 - 0.00370760743842 GOVTEXP + 0.033759450891\ GOVTREV - 0.318735158723\ INF - 2.04048296501 PRI \qquad (19)$$

with $R^2 = 0.899111$ and Dublin Watson Statistic (DW) = 1.48026

$$GOVTEXP = 130.623724121 + 0.48104158766\ GOVTREV - 0.796687861493\ INF - 2.47736930738\ PRI - 0.960150482021\ EXCH \qquad (20)$$

with $R^2 = 0.86712$ and Dublin Watson Statistic (DW) = 1.6765

$$GOVTREV = -1330.99654555 + 5.22260053639\ INF + 49.3707630333\ PRI + 14.5559437223\ EXCH + 0.800906942862\ GOVTEXP \qquad (21)$$

with $R^2 = 0.89346$ and Dublin Watson Statistic (DW) = 1.5762

$$INF = 42.0057801301 - 0.672814074065\ PRI - 0.209821955582\ EXCH - 0.00202517985786\ GOVTEXP + 0.00797375291363\ GOVTREV \qquad (22)$$

with $R^2 = 0.87635$ and Dublin Watson Statistic (DW) =1.47802

$$PRI = 25.4047831009 - 0.21136941388\ EXCH - 0.000990955980372\ GOVTEXP + 0{,}0118613471117\ GOVTREV - 0.1058725269\ INF \qquad (23)$$

with $R^2 = 0.857056$ and Dublin Watson Statistic (DW) = 1.56026

Vector Autoregressive (VAR) and Multiple Regression (MR) models obtained were used to obtain out sample forecast from 2020 to 2024. Table 5 and Table 6 were used to present the out sample forecast values obtained from both VAR and MR models. The results of both models indicated that Government revenue and Government expenditure indicate a continuous rise while Inflation rate, exchange rate and price of crude oil fluctuate from the year 2020 to 2024. Table 7 was used to display the forecast evaluations results based on root mean square forecast error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE). The values of the out-sample forecast for VAR models is better than MR models since the values of the forecast evaluation metrics were smaller in VAR forecast evaluation metrics. In essence, the nature of the model and forecasting method can have important implications in forecast evaluation performance.

***Table 5.* Out-sample forecast using vector autoregressive models**

Year(s)	Government Revenue in Billion(s) naira	Government Expenditure in Billion(s) naira	Inflation Rates in (%)	Exchange rate	Price of Crude oil

2020	3 930.50	1 456.20	16.001	130	38.4
2021	5 647.50	1 722.10	17.956	138	55.43
2022	5 985.10	1 978.00	8.518	141,8	68.39
2023	5 827.50	2 650.90	5.513	145	75.7
2024	7 886.59	3 270.82	11.781	128	97.84

Table 6. **Out-sample forecast using multiple regression analysis models**

Year(s)	Government Revenue in Billion(s) naira	Government Expenditure in Billion(s) naira	Inflation Rates in (%)	Exchange rate	Price of Crude oil
2020	3 920,50	1 426,20	15,001	133	38,3
2021	5 547,50	1 822,10	17,856	136	54,43
2022	5 965,10	1 938,00	8,218	131,8	65,39
2023	5 727,50	2 450,90	5,413	125	72,7
2024	7 866,59	3 240,82	11,581	120	97,64

Table 7. **Forecast evaluations for vector autoregressive and multiple regression model**

Forecast Evaluation	Vector Autoregressive Model	Multiple Regression Analysis Model
Exchange rate		
Root Mean Square Error	0.077303	0.158090
Mean Absolute Error	0.064060	0.112486
Mean Absolute Percent Error	1.394270	2.781109
Government expenditure		
Root Mean Square Error	0.080538	0.093224
Mean Absolute Error	0.063062	0.063062
Mean Absolute Percent Error	1.318467	1.358467
Inflation		
Root Mean Square Error	0.291060	0.302457
Mean Absolute Error	0.232815	0.248473
Mean Absolute Percent Error	21.08708	21.91332
Government revenue		
Root Mean Square Error	0.172531	0.172975
Mean Absolute Error	0.106115	0.106448
Mean Absolute Percent Error	2.025383	2.072488
Price of Crude oil		
Root Mean Square Error	0.182531	0.194975
Mean Absolute Error	0.206415	0.407148
Mean Absolute Percent Error	3.027683	4.086388

Conclusion

The time plot of the period under consideration indicated that each year there is a simultaneous increase in the values of government revenue, government expenditure and exchange rate except a sharp downward shift in government revenue in some years. Inflation rate fluctuates from year to year, cyclical in movement and a cycle is completed every three years. Crude oil price exhibited a continuous rise with a sudden rapid fall in 2014 to 2016 before a steady rise in late 2016. The Augmented Dickey-Fuller test was used to show that all the series were stationary at the first differencing, I(1) and only inflation rate was stationary at the ordinary level, I(0).

Vector Autoregressive (VAR) and Multiple Regression (MR) analysis models were used to show that there exist inter and linear relationships among all the macroeconomic variables but the relationship were better shown with *VAR* models. The out-sample forecast for both models indicated that Government revenue and Government expenditure exhibited a continuous rise while Inflation rate, exchange rate and price of crude oil fluctuate from the year 2020 to 2024. The forecast evaluations results based on Root mean square forecast error (RMSE), Mean absolute error (MAE) and Mean absolute percentage error (MAPE) showed that the out-sample forecast for Vector Autoregressive (VAR) model is better than Multiple Regression (MR) analysis models since the values of the forecast evaluation metrics were smaller in VAR forecast evaluation metrics. In essence, the nature of the model and forecasting method can have important implications in forecast accuracy and evaluation performance.

Conflict of interest

The authors declare no conflict of interest, financial or otherwise

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