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ANALYSIS AND MODELLING OF TRAFFIC FLOW ON NIGERIA HIGHWAYS

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Abstract: Traffic congestion is a condition on road networks that occurs as users increases, and is characterized by slower speeds, longer trip times, and increased vehicular queuing. The common example is the congestion of vehicles on roads. In this work the continuous and the discontinuous behaviour of vehicles is model based on Lighthill, Whitham and Richard (LWR) model, considering the traffic flow parameters such as flow, density and velocity.

Keywords: Analysis, Modelling, Traffic flow, Nigeria highways

Introduction

Mathematical model usually describes a system by set of variables and equations that establish relationship between the variables and these variables could be real numbers or integers. These variables represent some properties of the system, for example, event occurrence (yes/no). In an attempt to solve these daily challenges on the roadways, a traffic flow model was developed to help the transportation engineer to understand and express the property of traffic flow on the highways.

Traffic flow is the movement of vehicles along a road or street continuously at a particular time. Traffic flow model describes a precise mathematical way of how groups of vehicles at a particular time move, interact and how their movements are being affected by density, spotlights and other infrastructures on the road ways. Whether the task is to evaluate the capacity of existing roadways or design new roadways, most transportation engineering projects begin with an evaluation of traffic flow.

This paper aims to model the traffic flow of vehicles in a genuine urban dynamic traffic situation of long traffic congestion on the limited number of road on the peak hours.

Greenshields (1935) firstly proposed the traffic stream theory addressing the relationships among flow rate, speed, and density, in which speed and density are assumed to be linearly correlated. Greenberg (1959) revised the model of the speed and density to fit a logarithmic curve, based on a hydrodynamic analogy and assumption regarding the traffic flow as a perfect fluid and one dimensional compressible flow. Underwood (1961) used exponential expression for such a model. Researchers have disclosed the discontinuities of the relationships between traffic variables. Edie (1961) quantified the linear relationship between density and the logarithm of velocity above the optimum velocity for uncongested traffic and velocity and the logarithm of spacing (the inverse of density) for congested traffic. Multiple curves are often applied to depict the discontinuities. For instance, Koshi (1983) proposed a reverse lambda shape to describe the flow–density relationship. May (1990) developed the “two-regime” models to describe the relationship of flow and density. Hall (1986) proposed an inverted-V shape to represent the flowoccupancy relationship. Polus and Pollatschek (2002) proposed three regimes of traffic flows (free, dense, and unstable flows), and traffic breakdown was explained as the change from dense flow to unstable flow.

Kerner and Konha¨user (1994) and Kerner and Klenov (2010) defined traffic flows in three categories: free flow, synchronized flow, and stop-and-go flow. The free flow has high travel speed and low traffic volume and density. The congested traffic flow is further classified into synchronized flow (S) and wide moving jam (J). The synchronized flow has relative low speed and high volume and density. A wide moving jam is a moving jam that maintains the mean velocity of the downstream front of the jam as the jam propagates. They also disclosed the double Z-characteristic shape for relating speed and density. The empirical double Z-characteristic shape is used to depict the phase transitions between two different phases. $F \rightarrow S$ (free flow to synchronized flow) and $S \rightarrow J$ (synchronized flow to jam flow) transitions can be illustrated by a double Z shape (or termed Z-characteristic) for the $F \rightarrow S \rightarrow J$ (free to synchronized to jam conditions) transitions. The double Z-characteristic consists of a Z-characteristic for an F S transition and a Z-characteristic for an $S \rightarrow J$ transition, as well as the phases associated with the critical speeds required for the phase transitions. The synchronized traffic defined by Kerner is also described as the traffic oscillation by other researchers. Treiber and Kesting (2011) studied the convective instability in congested traffic flow, and they classified congested traffic flow into five classes according to the stability that lead to significantly different sets of traffic patterns (2013).

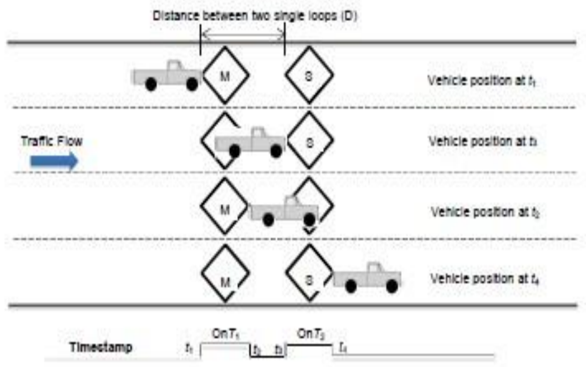

Figure 1: Layout of a dual-loop detector on highway.

Materials and Methods

Assumptions of the Traffic Flow Theory

We now introduce additional assumptions which are supported by both theoretical and experimental findings:

i. Measuring the velocity and position of each individual’s car on the road is too difficult.So, this model views the distribution of cars by looking at the density of the cars which is the number of cars per mile on the road. We assume that the density is the only property of the cars which matters.

ii. The second assumption follows from the first. Only the density of the cars matters.

iii. Therefore the average velocity of the cars at any point depends on the density of the

cars.
According to these assumptions, the flow rate q depends on x and t only through the density, that is, $q = Q(\rho)$ for some function Q this relation seems to be reasonable in the sense that the density of cars surrounding a given indeed controls the speed limits, weather conditions, and road characteristics.
We consider here a particular $q = \rho v$ where v is the average local velocity of cars

$$\frac{\partial \rho}{\partial t} + C(\rho)\frac{\partial \rho}{\partial x} = 0 \quad (1)$$

Where $c(\rho) = q^0(\rho) = v(\rho) + \rho v(\rho)$
In general, the local velocity $V(\rho)$ is a increasing function of ρ, where $V(\rho)$ has finite maximum value V_{max} at $\rho = 0$ and decrease to zero at $\rho = \rho_{max} = \rho_m$. Or the value of $\rho = \rho_m$ the cars are bumper to bumper. Since $q - \rho V$, $q(\rho) = 0$ when $\rho = 0$ and $\rho = \rho_m$ this means that q is an increasing function of ρ until it attains maximum value $q_{max} = q_m$ and some $p = p_m$ and then decrease to zero at $\rho = rho_m$. With the wave propagation velocity, (2) becomes

$$c(\rho) = V(\rho) + \rho V^0(\rho) \quad (3)$$

Model Formulations
In this model the total number of vehicles moving on a road $N(t)$ is classifies into four compartments namely., Free vehicles F(t), Slow vehicles $S(t)$, Blocked vehicles $B(t)$, and Discharged vehicles $D(t)$. So that

$$N(t) = F(t) + S(t) + P(t) + D(t) \quad (4)$$

Equation (4) is interpreted as,

1. $N(t)$ denotes total population size of vehicles under consideration
2. $F(t)$ denotes the population size of Free Vehicles that are flowig freely without any influence of blocking effects but there is a possibility to face blocking in future
3. $S(t)$ denotes the population size of slow vehicles which are partially blocked which are moving under the influence of blockings
4. $B(t)$ denotes the population size of blocked vehicles which are totally blocked and are almost stopped and
5. $D(t)$ denoted the population size of Discharged vehicles which are just released from blocking and have a chance of experiencing blockings again.

The model assumptions are taking into consideration when describing the rate of change of free vehicles on a road.

1. The rate of free vehicles increase by a constant rate τ is just similar to the constant birth rate in epidemiology
2. Vehicles are assumed to be blocked by blocked vehicles at a rate of α. This parameter α is just similar to the transfer rate of infection in epidemiology
3. Vehicles are assumed to be released from blockage at a rate of r_2. This parameter r_2 is just similar to the recovery rate from infection in epidemiology
4. Let μ_d be the rate of vehicles leaving from the present road to follow another road. This means that the vehicles in each compartment decreases when the vehicles change the road. Thus, μ_d is just similar to natural death rate in epidemiology.

Thus, the rate of change of free vehicles on the road is given as:

$$\frac{dF}{dt} = \tau - \alpha FB + r_2 D - \mu_d F \quad (5)$$

The rate of slow vehicles is assumed to increase due to blocking of free vehicles at the rate of α and blocking of discharged vehicles at the rate of δ. But, this rate is assumed to decrease by discharging of slow vehicles at rate γ and blocking of slow vehicles at rate
η. Thus the rate of change of slow vehicles on a road is given by

$$\frac{dS}{dt} = \tau - \alpha FB + r_2 D - \mu_d F \quad (6)$$

The rate of Slow vehicles is assumed to increase due to blocking of free vehicles at the rate of α and blocking of discharged vehicles at the rate δ. But, this rate is assumed to decrease by discharging of slow vehicles at rate ofγ and blocking of slow vehicles at rate η. Thus, the rate if change of Slow vehicles on a road is given by

$$\frac{dS}{dt} = \alpha + \delta D - \gamma S - \eta S - S\mu_d \quad (7)$$

The rate of blocked vehicles is assumed to increase at a rate of η due to slow vehicles but decrease at the rate of r_1 due to discharging of blocked vehicles. Hence the rate of change of blocked vehicles on a road is given by

$$\frac{dB}{dt} = \eta S - r_1 B - B\mu_d \quad (8)$$

The rate of change of discharged vehicles is assumed to increase by discharging of blocked vehicles at the rate r_1 and discharging of slow vehicles at the rate γ. However, this rate is assumed to decrease at the rate of δ due to blocking of discharged vehicles and also at the rate r_2 due to releasing of vehicles from blockage. Thus, the rate of change of discharged vehicles is given by

$$\frac{dD}{dt} = Br_1 + \gamma S - r_2 D - \delta D - D\mu_d \quad (9)$$

Table 1: Description of Model Parameter

Parameter	Description Pertaining to traffic flow
τ	Rate of new vehicles joining the road (OR) growth rate of free vehicles
α	Rate of free vehicles gradually becoming slow vehicles
η	Rate of slow vehicles becoming blocked vehicles
r_1	Rate of Blocked vehicles becoming discharged vehicles
γ	Rate of slow vehicles becoming discharged vehicles
δ	Rate of discharged vehicles becoming slow vehicles
r_2	rate of discharged vehicles becoming free vehicles
μ_d	Rate of vehicles following another routes.

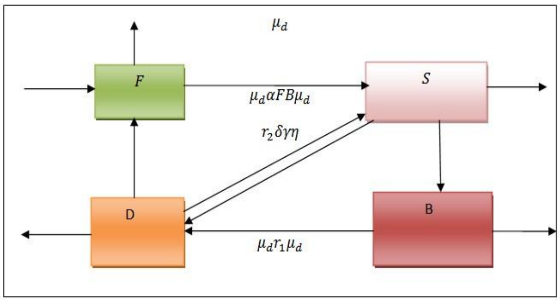

Figure 2: Progression of vehicles among Free (*F*), showed (*S*), Blocked (*B*), and Discharged (*D*)

The nonlinear differential equations of the dynamical traffic flow using table 1 and figure 2 are:

$$\frac{dF}{dt} = \lambda - \alpha FB + r_2 D - \mu_d F \quad (10)$$

$$\frac{dS}{dt} = \alpha FB + \delta D - \gamma S - \eta S - \mu_d S \quad (11)$$

$$\frac{dB}{dt} = \eta S - r_1 B - \mu_d B \quad (12)$$

$$\frac{dD}{dt} = Br_1 + \gamma S - r_2 D - \delta D - \mu_d D$$

(13)

with initial conditions $F(0)$, $S(0)$, $B(0)$ and $D(0)$.

Derivation of Basic Retardation Number

Basic retardation number is the average number of slow or blocked vehicles generated by each blocked vehicle. Calculating retardation number is important to analyze the local stability of nonlinear system of equations (10)-(13) The retardation number is the largest eigenvalue of matrix $K = FV^{-1}$.

Where

$$\mathcal{F} = \left(\frac{\partial f}{\partial x_j}\right)|_{E_0}$$

$$V = \left(\frac{\partial v}{\partial x_j}\right)|_{E_0} \quad (14)$$

Here, f is the newly blocking terms and v is non-singular matrix of the remaining transfer terms. Now, the basic retardation number R_0 of the model (10)-(13) is computed using the next

generation matrix in similar procedure as reproduction number in epidemiological concept used to be computed. Thus, the next generation matrices $f, v, \mathcal{F}, V, V^{-1}\ \mathcal{F}V^{-1}$ are constructed respectively as follows:

$$f = \begin{bmatrix} \alpha FB \\ 0 \end{bmatrix}$$

$$v = \begin{bmatrix} -\delta D + (\mu_d + \gamma + \eta)S \\ -\eta + \beta r_1 + \mu_d B \end{bmatrix}$$

$$\mathcal{F} = \begin{bmatrix} 0 & \frac{\alpha\tau}{\mu_d} \\ 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} \gamma + \eta + \mu_d & 0 \\ -\eta & r_1 + \mu_d \end{bmatrix}$$

$$V^{-1} = \begin{bmatrix} \frac{1}{(\gamma+\eta+\mu_d)} & 0 \\ \frac{\eta}{[\mu_d(r_1+\mu_d)(\gamma+\eta+\mu_d)]} & \frac{1}{(r_1+\mu_d)} \end{bmatrix}$$

$$K = \mathcal{F}V^{-1} = \begin{bmatrix} \frac{\alpha\lambda\eta}{[\mu_d(r_1+\mu_d)(\gamma+\eta+\mu_d)]} & \frac{\alpha\tau}{(r_1+\mu_d)\mu_d} \\ 0 & 0 \end{bmatrix}$$

Clearly, eigenvalues of next generation matrix K are $\lambda_1 = 0$ and $\lambda_2 = \frac{\alpha\lambda\eta}{[\mu_d(r_1+\mu_d)(\gamma+\eta+\mu_d)]}$

among which λ_2 is larger, Hence, the retardation number of the model is given by

$$R_0 = \rho(\mathcal{F}V^{-1}) = \left\{\frac{\alpha\lambda\eta}{[\mu_d(r_1+\mu_d)(\gamma+\eta+\mu_d)]}\right\} \quad (15)$$

Equilibrium Points

An equilibrium solution is a steady state solution of the model equations (10)-(13) in the sense that if the system begins at such a state, it will remain there for all times as long as any disturbance occurs. In other words, the population sizes remain unchanged and thus the rate of change for each population vanishes. Equilibrium points of the model are found, categorized, stability analysis is conducted and the results have been presented in the following:

Blocking Free Equilibrium

At blocking free equilibrium vehicles flow feely without any interference of any kind of blocking, That is, at this equilibrium vehicles we will run freely with speeds as per the wish of their drivers. Furthermore, at this equilibrium no vehicle is forced either to run with slower speeds or to stop completely. That is $S = B = 0$. Thus under this assumption the system of equation (10)-(13) reduce to the following form

$$\frac{dF}{dt} = \lambda + r_2 D - \mu_d F = 0$$

$$\delta D = 0$$

$$\frac{dD}{dt} = -r_2 D - \delta D - \mu_d D = 0$$

Further, the solution of these reduced form of equations can be obtained with simple algebraic operation as $D = 0$ and $F = \frac{\tau}{\mu_d}$. Here, $D = 0$ implies that no car is released

from blockings. In fact, there is no blocking here. Thus, blocking free equilibrium BFE of the model is obtained as

$$E_0 = \left(\frac{\tau}{\mu_d}, 0,0,0\right)$$

The Endemic Equilibrium Point

Let $X^* = (F^*, S^*, B^*, D^*)$ be an endemic equilibrium point. In order to obtain endemic equilibrium E^* point of the model the left hand sides of the equations (10)-(13) are set equation to zero. Thus, the model equations reduce to the form as

$$\tau - \alpha F^*B^* + r_2D^* - \mu_d F^* = 0$$

$$\alpha F^*B^* + \delta D^* - \gamma S^* - \eta S^* - \mu_d S^* = 0 \quad (16)$$

$$\eta S^* - r_1 B^* - \mu_d B^* = 0$$

$$B^* r_1 + \gamma S^* - r_2 D^* - \delta D^* - \mu_d D^* = 0$$

The endemic equilibrium is the solution of the set of equations (16). On employing simple algebraic manipulations the solution can be as

$$F^* = \frac{\tau}{\mu_d} \quad B^* = 0 \qquad D^* = 0 \qquad S^* = 0 \quad (17)$$

Hence, the endemic equilibrium point of the model is given by $X^* = \left(\frac{\lambda}{\mu_d}, 0,0,0\right)$. This

Shows the only equilibrium point is the blocking free equilibrium point.

Stability Analysis of the Blocking Free Equilibrium

In the absence of blocking, the traffic flow model will have a unique blocking free steady state E_0. To find the local stability of E_0, the Jacobian matrix of the model equations valued at blocking free equilibrium point E_0 is used. It is already shown that the BFE of model (10)-(13) is given by $E_0 = \left(\frac{\tau}{\mu_d}, 0,0,0\right)$.

Now, the stability analysis of BFE is conducted and the results are presented in the form of theorems and proofs in the following.

Local Stability of Blocking Free Equilibrium

Let J be the Jacobian matrix formed from system of equations (10)-(13). Thus, following the procedures given in the literature, local stability of blocking free equilibrium point is found. Now,the Jacobian matrix that is constructed from the model (10)-(13) is

$$\mathcal{J}(F,S,B,D) = \begin{bmatrix} -\alpha B - \mu_d & 0 & -\alpha F & r_2 \\ \alpha B & -(\gamma+\eta+\mu_d) & \alpha F & \delta \\ 0 & \eta & -(r_1+\mu_d) & 0 \\ 0 & \gamma & r_1 & -(r_2+\delta+\mu_d) \end{bmatrix}$$

(18)

Similarly, the Jacobian matrix J reduces to the form, at the blocking free equilibrium point, as

$$\mathcal{J}\left(\frac{\tau}{\mu_d}, 0,0,0\right) = \begin{bmatrix} -\mu_d & 0 & -\frac{\alpha\tau}{\mu_d} & r_2 \\ 0 & -(\gamma+\eta+\mu_d) & \frac{\alpha\tau}{\mu_d} & \delta \\ 0 & \eta & -(r_1+\mu_d) & 0 \\ 0 & \gamma & r_1 & -(r_2+\delta+\mu_d) \end{bmatrix}$$

Now,

(i) Trace of $\mathcal{J}\left(\frac{\tau}{\mu_d}, 0,0,0\right) = -\mu_d - (\gamma+\eta+\mu_d) - (r_1+\mu_d) - (r_2+\delta+\mu_d)$
$= [4\mu_d + \gamma + r_1 + \eta + r_2 + \delta < 0]$

That is considering positive parametric values we can conclude that the trace of Jacobian matrix at disease free equilibrium point is negative.

(ii)$det\left(\frac{\tau}{\mu_d},0,0,0\right) =$

$$\begin{bmatrix} -\mu_d & 0 & -\frac{\alpha\tau}{\mu_d} & r_2 \\ 0 & -(\gamma+\eta+\mu_d) & \frac{\alpha\tau}{\mu_d} & \delta \\ 0 & \eta & -(r_1+\mu_d) & 0 \\ 0 & \gamma & r_1 & -(r_2+\delta+\mu_d) \end{bmatrix}$$

$= \gamma\mu_d^3 + r_2\mu_d^3 + \delta\mu_d^3 + \eta\mu_d^3 + r_1\mu_d^3 + \mu_d^4 + \delta r_1\mu_d^2 + \eta r_1\mu_d^2 + \gamma r_2\mu_d^2 + r_2\eta\mu_d^2 + \gamma r_1\mu_d^2 + r_1 r_2\mu_d^2 + \delta\eta\mu_d^2 + \delta\eta\alpha\tau - \eta\alpha\tau\mu_d + \gamma r_2 r_1\mu_d - r_2\eta\alpha\tau + r_2\eta r_1\mu_d]$

$= [\mu^4{}_d + (\gamma + r_2 + \delta + \eta + \eta + r_1)\mu^3{}_d + (\delta r_1 + \eta r_1 + \gamma r_2 + r_2\eta + \gamma r_1 + r_1 r_2 + \delta\eta)\mu^2{}_d + (\eta\alpha\tau + \gamma r_2 r_1 + r_2\eta r_1\mu_d - \eta\alpha\tau\delta + r_2)]$

Let $a_1 = [\mu^4{}_d + (\gamma + r_2 + \delta + \eta + \eta + r_1)\mu_d{}^3 + (\delta r_1 + \eta r_1 + \gamma r_2 + r_2\eta + \gamma r_1 + r_1 r_2 + \delta\eta)\mu^2{}_d + (\eta\alpha\tau + \gamma r_2 r_1 + r_2\eta r_1\mu_d - \eta\alpha\tau\delta + r_2)]$.

Now we can observe that determinant of Jacobian matrix $\mathcal{J}\left(\frac{\tau}{\mu_d},0,0,0\right)$ at blocking free equilibrium point is positive provided that $a_1 > a_2$.

Hence by Hourth Ruth theorem we can conclude that all negative all eigenvalues of Jacobian matrix are negative at blocking free equilibrium point. Further, using it can be concluded that blocking free equilibrium point is locally asymptotically stable for $R_0 < 1$ and unstable for $R_0 > 1$

Global Stability of Blocking Equilibrium

Let blocking expression be written as the following from which we state the nest theorem for global stability

$$x'^{(t)} = -(v - \mathcal{F})x(t) - \begin{pmatrix}\eta(F_0 - F)B \\ 0\end{pmatrix}$$

$$\frac{dF}{dt} = \tau - \alpha FB + r_2 D - \mu_d F$$

$$\frac{dD}{dt} = Br_1 + \gamma S - r_2 D - \delta D - \mu_d D \qquad (19)$$

Results and Discussion

Results

The following numerical values are assumed values allotted to parameters and variables used in the model and are used to describe the blocking effects on flow of vehicles. This simulation study describes the blocking effect on the motions of vehicles of all four categories on a road. The simulated vehicles flows have been observed over twenty minutes. The numerical simulation of the model has being carried out using a computer package (MATLAB).

Table 2: Values allotted to the parameters of the model

Parameter	τ	α	η	r_1	γ	δ	r_2	μ_d
Value	50	0.04	0.0001	0.4	0.6	0.004	0.4	0.15

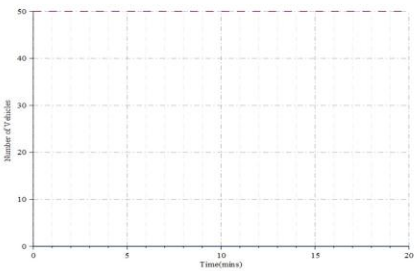

Figure 3: Numerical simulation of Free vehicles in the absence of blocking, inflow and outflow

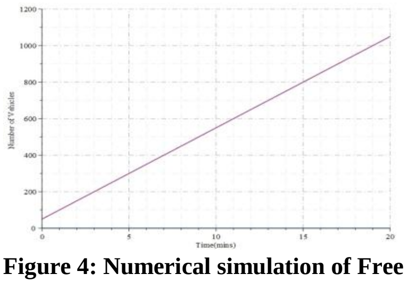

Figure 4: Numerical simulation of Free vehicles in the absence of blocking and outflow but with Inflow

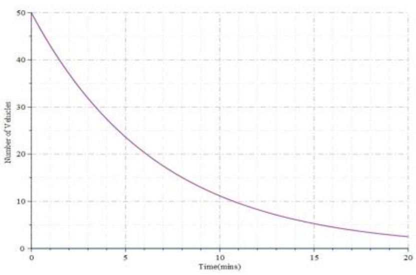

Figure 5: Numerical simulation of Free vehicles in the absence of Blocking and Inflow but with Outflow

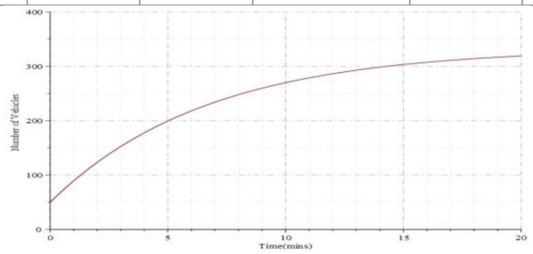

Figure 6: Numerical simulation of Free vehicles in the absence of Blocking but with Inflow and Outflow

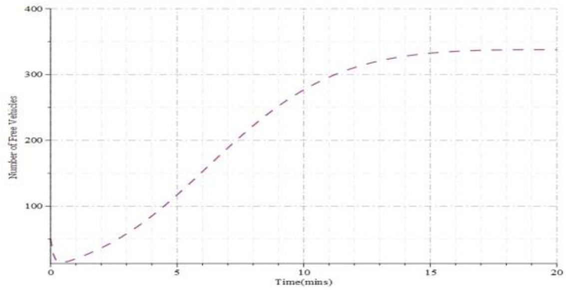

Figure 7: Numerical simulation of Free Vehicles in the presence of Blockings, Inflow and Outflow

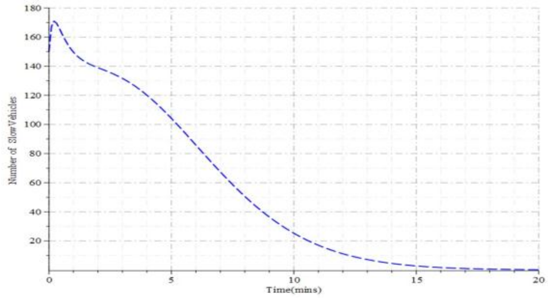

Figure 8: Numerical simulation of Slow Vehicles in the presence of Blockings, Inflow and Outflow

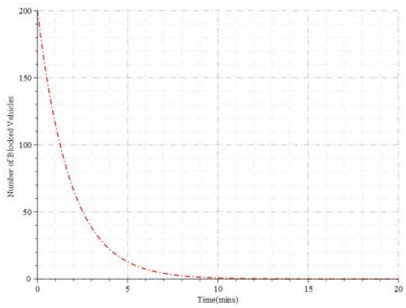

Figure 9: Numerical simulation of Blocked Vehicles in the presence of Blockings, Inflow and Outflow

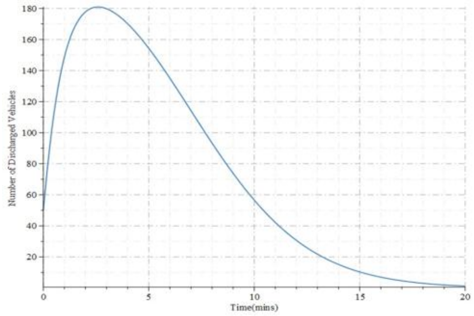

Figure 10: Numerical simulation of Discharged Vehicles in the presence of Blockings, Inflow and Outflow

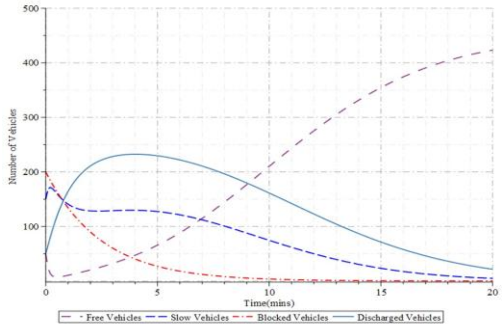

Figure 11: Numerical simulation of vehicles with Blockings, in the absence of Inflow and Outflow

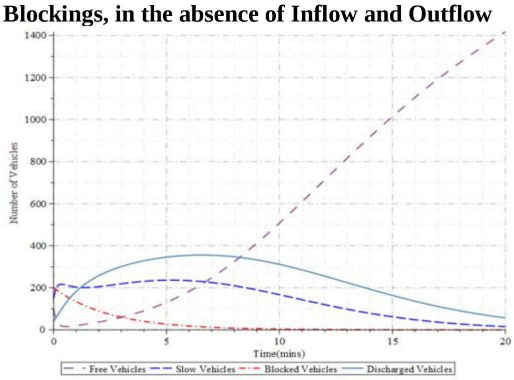

Figure 12: Numerical simulation of vehicles with Blockings and Inflow but in the absence of Outflow

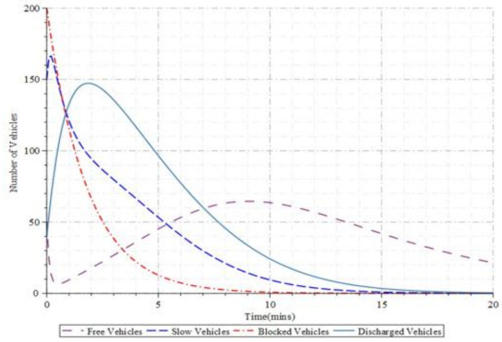

Figure 13: Numerical simulation of vehicles with Blockings and Outflow but in the absence of Inflow

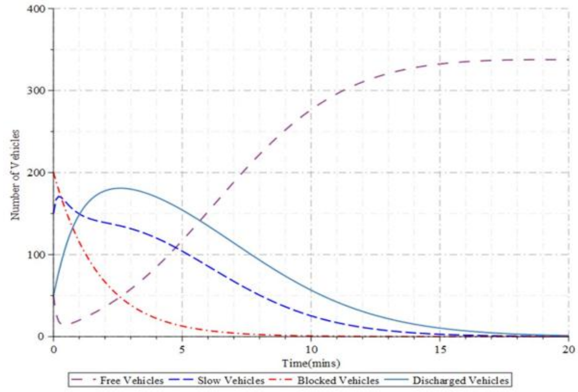

Figure 14: Numerical simulation of vehicles in the presence of Blockings, Inflow and Outflow

Discussion

It is observed that in Fig 3 the population size of the Free vehicles remain constant at F = 50 in absence of Inflow, Outflow and Blockings of vehicles. In addition, it can also be seen that as there are no blockings on the road all vehicles can move freely at any speeds. Hence, 50 vehicles flow freely from the beginning to the end of the road.

Figure 4 depicts the numerical simulation of free vehicles in the absence of blocking and outflow but with inflow. The graphical representation in Figure 2 is achieved by using the parametric values given in Table 2 together with $F(0) = 50$, $S(0) =$

0, $B(0) = 0$, $D(0) = 0$, $\tau = 40$, $\mu_d = 0$. It can be seen that the population size of the Free vehicles grow linearly from the initial size $F = 50$ and increases to about $F = 1050$ in absence of Outflow and Blockings of vehicles. However, inflow of the vehicles is allowed. That is, as time increases the number of freely flowing vehicles increases linearly from the beginning to the end of the road because there are no blockings.

Figure 5 illustrates the numerical simulation of free vehicles in the absence of blocking and inflow but with outflow. The graphical representation in Figure 3 is gotten by using the parametric values given in Table 2 together with $F(0) = 50$, $S(0) = 0$, $B(0) = 0$, $D(0) = 0$, $\tau = 0$, $\mu_d = 0.15$. It is found that the population size of the Free vehicles decreases from the initial size $F(0) = 50$ to $F = 0$ in absence of Inflow and Blockings of vehicles but with outflow. That is, in presence of outflow together with no inflow and no blockings, the number of freely moving vehicles decreases exponentially till the road becomes empty of vehicles.

Figure 6 shows numerical simulation of free vehicles in the absence of blocking but with inflow and outflow. The graphical representation in Figure 4 is obtained by using the parametric values given in Table 2 together with $F(0) = 50$, $S(0) = 0$, $B(0) = 0$, $D(0) = 0$, $\tau = 50$, $\mu_d = 0.1$. It is observed that the population size of the Free vehicles increases from the initial size $F(0) = 50$ to the upper bound $F = 320$ in absence of

Blockings of vehicles but with Inflow and outflow. That is, in presence of inflow and outflow together with no blockings, the number of freely moving vehicles increase exponentially till the number of vehicles on the road reaches its upper bound $F = 320$.

Figure 7 displays numerical simulation of free vehicles with the presence of blocking, inflow and outflow. The graphical representation in Figure 5 is gotten by using the parametric values given in Table 2 along with $F(0) = 50$, $S(0) = 150$, $B(0) = 200$, $D(0) = 50$, $\mu_d = 50$, $\mu_d = 0.15$. It is discovered that in presence of Inflow, outflow and Blockings of vehicles, the population size of Free vehicles initially decreases from the initial size $F(0) = 50$ to a minimum value and then increases.

Figure 8 shows numerical simulation of slow vehicles with the presence of blocking, inflow and outflow. The graphical representation in Figure 6 is obtained by using the parametric values given in Table 2 alongside $F(0) = 50$, $S(0) = 150$, $B(0) = 200$, $D(0) = 50$, $\tau = 50$, $\mu_d = 0.15$. It is observed in presence of Inflow, outflow and Blockings of vehicles, the population size of slow vehicles initially increases from the initial size $S(0) = 150$ to a maximum value and then decreases over time. Also, it can be noted that as there are inflows, outflows, and blockings with given initial conditions slow vehicles increases because of the blocking of restricting the speed of the vehicles. But the number of slow vehicles decreases as the road becomes free because of outflow

Figure 9 represents numerical simulation of blocked vehicles with the presence of blocking, inflow and outflow. The graphical representation in Figure 7 is obtained by using the parametric values given in Table 2 together with $F(0) = 50$, $S(0) = 150$, $B(0) = 200$, $D(0) = 50$, $\tau = 50$, $mu_d = 0.15$. It is found that in the presence of Inflow, outflow and Blockings of vehicles, the population size of blocked vehicles decreases from the initial size $B(0) = 200$ to zero i.e $B = 0$. As well, it can be said that as there are inflows, outflows, and blockings with the given initial conditions the number of blocked vehicles gets decreased exponentially, reason attributed to the fact that as time increases most vehicles move freely.

Figure 10 demonstrates numerical simulation of discharged vehicles with the presence of blocking, inflow and outflow The graphical representation in Figure 8 is obtained by using the parametric values given in Table 2 along with $F(0) = 50$, $S(0) = 150$, $B(0) = 200$, $D(0) = 50$, $\tau = 50$, $\mu_d = 0.15$. The figure shows that in the presence of Inflow, outflow and Blockings of vehicles, the population size of Discharged vehicles increases from the initial size $D(0) = 50$ to a maximum value and from there monotonically decreases over time. In addition, it can be interpreted that as there are inflows, outflows, and blockings with the given initial conditions discharged vehicles increases as vehicles are released from blockings but in presence of outflow and increment of freely moving vehicles the number of discharged vehicles decrease.

Figure 11 shows the numerical simulation of vehicles with blocking in the absence of inflow and outflow. The graphical representation of Figure 9 is obtained by using the parametric values given in Table 2 together with $F(0) = 50$, $S(0) = 150$, $B(0) = 200$, $D(0) = 50$, $\tau = 0$, $\mu_d = 0$. It is observed that: freely moving vehicles increase and then become stable because of the decrease in the number of vehicles discharged from all blockings; discharged vehicles increases and then decreases thereby releasing vehicles to move freely; the number of slow moving vehicles decreases as a result of increasing number discharged from blockings; and the number of blocked vehicles decreases as time increases. Consequently, all vehicles flow out of the road over time.

Figure 12 describes the numerical simulation of vehicles with blocking and inflow but no outflow. The graphical representation of Figure 10 is obtained by using the parametric values in Table 2 alongside with $F(0) = 50$, $S(0) = 150$, $B(0) = 200$, $D(0) = 50$, $\tau = 50$, $\mu_d = 0$. Results shows that: Free vehicles decrease because the road is overcrowded and as a result slowing free vehicle causing a decline in the number of vehicles moving freely and increases because of inflow of free vehicles and discharged vehicles; discharged vehicles increases as the result of more vehicles discharged from blockings; slow vehicles initially increases because of blockings, then decreases and finally increases as inflow Free moving vehicles and discharged vehicles are slowed down; blocked vehicles decreases gradually as more vehicles discharged and the road undergoing slow and discharging condition.

Figure 13 display the numerical simulation of vehicles with blocking and inflow but no outflow. The graphical representation of Figure 4.10 is obtained by using the parametric values in Table 2 alongside with $F(0) = 50$, $S(0) = 150$, $B(0) = 200$, $D(0) = 50$, $\tau = 0$, $\mu_d = 0.5$. Results shows that: free vehicles decrease because the road is overcrowded and as a result slowing the speed of free vehicle thereby causing a decline in the number of vehicles moving freely; discharged vehicles increases as the result of vehicles discharged from blockings and decreases as there are unceasing outflow of vehicles; slow moving vehicles increases as freely moving vehicles and discharged vehicles get slowed down because of blockings and decreases because of discharging and outflow; blocked vehicles decreases as there are discharging and outflow. Consequently, all vehicles go out of the road over time.

Figure 14 depicts the numerical simulation of vehicles with blocking and inflow but no outflow. The graphical representation of Figure 10 is obtained by using the parametric values in Table 2 alongside with $F(0) = 50$, $S(0) = 150$, $B(0) = 200$, $D(0) = 50$, $\tau = 50$, $\mu_d = 0.5$. It is found that: Free vehicles decrease because of crowdedness of the road results in blocking and increases as a result of inflow of free vehicles joining the road; discharged vehicles increases as a result of vehicles discharged from blocking and decreases as there are unceasing outflow of vehicles following another route; slow vehicles increases as free vehicles and discharged vehicles get slow down due to blockings and then decreases because of discharging and outflow of vehicles escaping from blocking; blocked vehicles decreases as there are discharging and outflow. Consequently, increasing number of free vehicles move freely on the road over time and the number of blocked vehicles decreases very fast.

Summary

Traffic flow theory comprises the study of the movement of individual drivers and vehicles between two points and the interactions they make with one another that plays a vital role in the progress of overall social productivity. Traffic congestion is a condition on road networks that occurs as use increases, and is characterized by slower speeds, longer trip times, and increased vehicular queuing. The most common example is the physical use of roads by vehicles. With the ever-increasing population growth and their demand for vehicles, traffic congestion has become a genuine problem, the challenge of traffic flow has motivated many researchers to model traffic flow at both the macroscopic and microscopic levels. This study seeks to model the continuous and the discontinuous behavior of vehicles by using the traffic flow parameters; flow, density and velocity with the use of partial differential Equations (PDEs) based on Lighthill, Whitham and Richard (LWR) model in order to help traffic engineers to verify whether traffic properties and characteristics such as speed(velocity), density and flow among others determines the effectiveness of traffic flow.

Conclusion

Our calculations enable us to conclude that, at low densities of the incoming traffic flow, ”sleeping policemen” enable the speed to be controlled in the required way along the sections where they are installed, without interfering with the free motion of the traffic. However, when the density of the incoming traffic flow increases, they produce a ”travelling jam”, which moves in the opposite direction to the traffic flow, which, in the final analysis, leads to congestion on the road. Control of the traffic using traffic lights enables one, by choosing the optimum mode of operation (the duration of the signals of different colour), to increase the throughout considerably.

The model takes into account the main property of traffic flows, namely, selforganization, and enables the conditions required to ensure maximum throughput, the occurrence and evolution of ”travelling jams” on roads, and the effect of the main components of traffic control, to be correctly described both qualitatively and quantitatively.

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