ON THE COMPARATIVE ANALYSIS OF ESTIMATION PROCEDURES

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Abstract

The linear estimation and the least squares estimation are known estimation procedures. This study tends to investigate the robustness of these procedures. To determine the robustness of linear and least squares estimation procedures, the correlation of the two procedures are determined. Thereafter, the power of estimation(P_{ca}) is applied to determine the robustness or accuracy of the estimation procedures using the correlation values and the sample size of the data set. Robustness of the estimation procedures is determined if $P_{ca} > 0.5$ otherwise if $P_{ca} < 0.5$ implies that the variation between the original data and the estimated data set is relatively large. The results revealed that the correlation and the value of P_{ca} demonstrated that the two estimation procedures are perfect. However, for this study, the average $P_{ca} = 0.735$ which means that estimated values are robust this concludes that the estimated values of the data set is robust for the linear and least square estimation procedures.

Keyword: Linear estimation; Least squares; Error; Power of estimation

Introduction

Linear estimation applies data set associated to different variables to compute the estimates (Bultheel and van Barel 1994)(Srivastava 2017). Defined the random variables $x_j, j = 1,2,3,\ldots,n$ as the independent random variable. This concept produces the must tractable prediction variable τ that depends on x_j. The assumption is that the tractable dependent variables and the independent variables x_j have joint normal distribution (Tagar and Vepa 2021; Raman and Sarkar 2016). Linear estimation is associated to the theory of estimation (Giacobello *et al.* 2012; de Frein 2021;Marple and Carey 1989) . The least square estimate like other estimation procedure has been discussed extensively in different fields (Jaeger 2006). The least square procedure can be linked to Galton (1886) though proposed by Legendre A. (1800s). Pearson and Fisher expanded Galton's initial concepts in diverse ways (Okwonu et al. 2021). The purpose of using the least square procedure is to estimate and fit the given data set into the function to obtain inliers after the influential observations has been identified. This can be described by considering pairs of observations $(x_j, y_j), j = 1,2,..,n,$ that is the independent variable x_j and the dependent variable y_j.The least square procedure minimize the variable by estimation which comprises the sum of squares deviation(Ahmad and Okwonu 2011; Okwonu and Othamn 2013; van de Geer 2005). This is used to determine the line of best fit of the data set (Miller 2006; Lindley, Moore, and McCabe 1999; Penenberg 2016).

Previous study mainly focusses on the values of the estimated data set to reduce the effects of outliers before other procedures are applied to the inlier data set. We extend this outlier detection procedure to investigate the relationship between the original data and the estimated data set. We further determine the relationship between by computing the correlation and further determine the robustness of the relationship between the original data and estimated data set.

This study is designed to apply the linear estimation and least squares procedures to estimates the data set and to determine the robustness of the estimated data set based on the correlation value. We proposed the estimation power procedure to determine the accuracy or robustness of the of the estimated data in comparison to the original data set.

The rest of this paper is described as follow. The estimation procedures are presented in Section 2. Results and analysis are contained in Section 3. Conclusion is contained in Section 4.

Methods

Linear estimation (LE)

The Linear estimation (LE) has been applied to compute robust estimates for variants of univariate classifiers (F. Z. Okwonu et al. 2022) This procedure can be described as

$$\Delta = \bar{\tau} + \frac{|\alpha,\theta|}{|\theta^2|} \times |\theta| = \bar{\tau} + \rho_{\tau x}\frac{\sqrt{\sigma_\tau^2}}{\sqrt{\sigma_x^2}} \times |\theta| \quad (1)$$

Where, $\nabla = 1, \theta = x - \hat{\bar{x}}, \hat{\bar{x}} = \mu = E(x),$

$E|\tau - \nabla|^2 = \bar{\tau}, \; and \; \alpha = \tau - E(\tau),$ τ denote the dependent random variable and $\rho_{\tau x} = \frac{\sigma_{\tau x}}{\sqrt{\sigma_{\tau\tau}\sigma_{xx}}}$, (Okwonu, Laro Asaju, and Arunaye 2020) denotes the correlation between τ and x, this implies2.0 that σ_τ^2 and σ_x^2 are the variance of τ and x respectively. The error of estimate is given as

$\pi = y - \Delta.$ (2)

From equation (2) we obtain the absolute estimation error

$|\pi| = |y - \Delta|$ (3)

Least Square estimation (LSE)

Similar to equations (1-3), the least square estimate based on the least squares procedure can be stated as

$\hat{y} = b + cx,$ (4)

Where x and $\hat{y}$ denotes the independent and dependent variables respectively, c denotes the slope, that is,

$$c = \frac{SS_{xy}}{SS_{xx}},$$

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n}, SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$\bar{x} = \frac{\sum_{j=1}^{n} x_j}{n}, \bar{y} = \frac{\sum_{j=1}^{n} y_j}{n},$$

and b intercept is given as $b = \bar{y} - c\bar{x}$. Based on the procedure in (Bultheel and van Barel 1994; Okwonu and Othman, 2013; Okwonu *et al.* 2021), the estimation error can be computed as

$e = y - \hat{y}.$ (5)

From the procedure in (Kern 2016), the absolute estimation error can be computed as follows.

$|e| = |y - \hat{y}|.$ (6)

The comparative between estimation errors from the linear estimation and the least square estimates can be expressed as

$|d| = |e - \omega|$ (7)

Equation (7) is applied to determine the absolute estimation error deviation between the two estimation procedures.

Power of estimation

The power of estimation simply measures the estimation accuracy or robustness between the given data and the estimated data based on the sample size. The power of estimation is computed using the estimates of the two methods in which case, the correlation coefficient is determined. The formula for the power of estimation is given as

$$P_{ca} = \frac{\left|\rho_{xy}^{\frac{1}{n}}\right|}{2\sqrt{|\rho_{xy}|}}, 0 \leq P_{ca} \leq 1 \quad (8)$$

where ρ_{xy} denoted the Pearson correlation . The P_{ca} is applied to determine the strength of estimation such that if $P_{ca} < (0.5)$ means that the estimation is relatively weak and if $P_{ca} = 0.5$ it means that the estimate is balanced due to equal data contribution with perfect correlation and $P_{ca} =$ 1implies perfect estimation in which case, equations (2,5) tends to zero and $P_{ca} > 1$ implies that the correlation value is extremely weak and in such case, determining the strength of estimation becomes problematic. This is due to a very large deviations in equations (2,5) for the case of between absolute estimation errors. From equation (8), we observed that P_{ca} depends on ρ_{xy} and the sample size (n). The sample size plays vital role in determining the robustness of the estimated values. The effect of the sample size on P_{ca} satisfies the concept of the central limit theorem.

Results and analysis

The data set used in this study is culled from (Okwonu et al. 2022). The data set consist of 100 measured (cm) cattle horns (bull and cow). The data set and computational results are contained in Table 1 and Table 2 respectively. In both cases, we estimated the cattle horn width (**bold**). From both tables, we observed that the estimated values is relatively close to horn width than the horn length for the bull similar output is replicated for the cow width. The respectively estimated errors are similar.

Table 1. Length and Base width measurements of Bull horns

Horn length (x)	**Horn width (y)**	$\hat{y}$	Δ	$e = y - \hat{y}$	$\pi = y - \Delta$	$\lvert d\rvert = \lvert e - \pi\rvert$
17.6	**8.6**	6.75121	6.76161	1.848793	1.838394	0.010399
21.5	**9.5**	8.45355	8.43825	1.046452	1.061755	0.015303
21	**7.8**	8.2353	8.22329	-0.435299	-0.423292	0.012007
17.2	**6.4**	6.57661	6.58964	-0.176608	-0.189643	0.013035
16.5	**8.2**	6.27106	6.28871	1.928941	1.911292	0.017649
21.9	**8.7**	8.62815	8.61021	0.071853	0.089792	0.017939
18.1	**9.8**	6.96946	6.97656	2.830545	2.82344	0.007105
11.7	**8.1**	4.17587	4.22515	3.92413	3.874849	0.049281
15.4	**7.2**	5.79091	5.81581	1.409088	1.384191	0.024897
18.2	**8.5**	7.01311	7.01955	1.486895	1.48045	0.006445
17.8	**7.4**	6.83851	6.84759	0.561494	0.552413	0.009081
16.5	**6.3**	6.27106	6.28871	0.028941	0.011292	0.017649
21.5	**8.8**	8.45355	8.43825	0.346452	0.361755	0.015303
22	**10.1**	8.6718	8.6532	1.428204	1.446801	0.018597
20.1	**9**	7.84245	7.83637	1.157549	1.163625	0.006076
21	**10.1**	8.2353	8.22329	1.864701	1.876708	0.012007

17.3	**7.2**	6.62026	6.63263	0.579743	0.567366	0.012377
19.2	**9.4**	7.4496	7.44946	1.950397	1.950542	0.000145
20.1	**10**	7.84245	7.83637	2.157549	2.163625	0.006076
18.2	**7.4**	7.01311	7.01955	0.386895	0.38045	0.006445
18	**8.2**	6.92581	6.93357	1.274194	1.266431	0.007763
17.6	**7**	6.75121	6.76161	0.248793	0.238394	0.010399
18.2	**7.3**	7.01311	7.01955	0.286895	0.28045	0.006445
16.5	**9.2**	6.27106	6.28871	2.928941	2.911292	0.017649
21.5	**11.6**	8.45355	8.43825	3.146452	3.161755	0.015303
23.6	**10.4**	9.37019	9.34105	1.029807	1.058949	0.029142
22.7	**10.9**	8.97734	8.95413	1.922655	1.945866	0.023211
24.2	**11.9**	9.63209	9.599	2.267909	2.301004	0.033095
22.4	**11**	8.8464	8.82516	2.153604	2.174838	0.021234
19.8	**8.6**	7.7115	7.7074	0.888498	0.892598	0.0041
21.2	**7.4**	8.3226	8.30927	-0.922598	-0.909273	0.013325
20.3	**6**	7.92975	7.92236	-1.92975	-1.922356	0.007394
19	**5.7**	7.3623	7.36348	-1.662303	-1.663476	0.001173
18.7	**5**	7.23135	7.2345	-2.231354	-2.234504	0.00315
19.4	**5.3**	7.5369	7.53544	-2.236902	-2.235439	0.001463
17.8	**4.7**	6.83851	6.84759	-2.138506	-2.147587	0.009081
19.8	**5.4**	7.7115	7.7074	-2.311502	-2.307402	0.0041
17.6	**4.2**	6.75121	6.76161	-2.551207	-2.561606	0.010399
18.6	**5.1**	7.1877	7.19151	-2.087704	-2.091513	0.003809
21	**6.5**	8.2353	8.22329	-1.735299	-1.723292	0.012007
17.7	**4.8**	6.79486	6.8046	-1.994856	-2.004597	0.009741
20.2	**6**	7.8861	7.87937	-1.886101	-1.879366	0.006735
19.6	**5.6**	7.6242	7.62142	-2.024202	-2.021421	0.002781
19.1	**5.1**	7.40595	7.40647	-2.305953	-2.306467	0.000514
17.9	**4.6**	6.88216	6.89058	-2.282156	-2.290578	0.008422
18.3	**5**	7.05676	7.06254	-2.056755	-2.062541	0.005786
17.9	**4.6**	6.88216	6.89058	-2.282156	-2.290578	0.008422
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17.9	**4.6**	6.88216	6.89058	-2.282156	-2.290578	0.008422
18.3	**5**	7.05676	7.06254	-2.056755	-2.062541	0.005786
17.9	**4.6**	6.88216	6.89058	-2.282156	-2.290578	0.008422
17.7	**4.8**	6.79486	6.8046	-1.994856	-2.004597	0.009741
20.2	**6**	7.8861	7.87937	-1.886101	-1.879366	0.006735
19.6	**5.6**	7.6242	7.62142	-2.024202	-2.021421	0.002781

Table 2. Length and Base width measurements of cow horns

Horn length (x)	**Horn width (y)**	$\hat{y}$	Δ	$e = y - \hat{y}$	$\pi = y - \Delta$	$\lvert d\rvert = \lvert e - \pi\rvert$
23.3	**10.3**	10.1271	10.2746	0.17292	0.02541	0.1475181
24.5	**8.8**	10.4607	10.5641	-1.6607	-1.7641	0.103461
25.6	**10.4**	10.7664	10.8295	-0.3664	-0.4295	0.063075
24.9	**9.2**	10.5718	10.6606	-1.3718	-1.4606	0.088775
29.3	**13.1**	11.795	11.7222	1.30504	1.37781	0.0727677
26.2	**9.2**	10.9332	10.9743	-1.7332	-1.7743	0.041047
25.9	**9**	10.8498	10.9019	-1.8498	-1.9019	0.052061
25	**8.9**	10.5996	10.6847	-1.6996	-1.7847	0.085104
26.5	**10.5**	11.0166	11.0466	-0.5166	-0.5466	0.030033
20.3	**9.1**	9.29314	9.5508	-0.1931	-0.4508	0.257661
23.7	**10**	10.2383	10.3711	-0.2383	-0.3711	0.132832
21.9	**9.5**	9.7379	9.93682	-0.2379	-0.4368	0.198918
27.2	**10.7**	11.2112	11.2155	-0.5112	-0.5155	0.004333
28.1	**12.2**	11.4614	11.4327	0.73862	0.76733	0.0287106
26.7	**11.9**	11.0722	11.0949	0.82779	0.8051	0.0226895
27.4	**11.3**	11.2668	11.2638	0.03321	0.03622	0.0030106
29.1	**17.4**	11.7394	11.6739	5.66064	5.72606	0.0654249
27.5	**12.6**	11.2946	11.2879	1.30541	1.31209	0.006682
29	**12.8**	11.7116	11.6498	1.08844	1.15019	0.0617535
27.3	**11.2**	11.239	11.2397	-0.039	-0.0397	0.000661
21.9	**11.1**	9.7379	9.93682	1.3621	1.16318	0.1989181
23.7	**10.4**	10.2383	10.3711	0.16173	0.0289	0.1328324
22.8	**13.8**	9.98809	10.154	3.81191	3.64604	0.1658752
24.9	**11**	10.5718	10.6606	0.42816	0.33938	0.0887753
26.2	**12.3**	10.9332	10.9743	1.36678	1.32574	0.0410466
29	**12.7**	11.7116	11.6498	0.98844	1.05019	0.0617535
30.1	**10.9**	12.0173	11.9152	-1.1173	-1.0152	0.10214
30.6	**17.1**	12.1563	12.0358	4.94367	5.06417	0.1204964
34	**11.2**	13.1015	12.8561	-1.9015	-1.6561	0.245325
26.2	**12**	10.9332	10.9743	1.06678	1.02574	0.0410466
30.1	**11.6**	12.0173	11.9152	-0.4173	-0.3152	0.10214
29.5	**11.3**	11.8506	11.7704	-0.5506	-0.4704	0.080111

27.4	**10**	11.2668	11.2638	-1.2668	-1.2638	0.00301
25.9	**9.8**	10.8498	10.9019	-1.0498	-1.1019	0.052061
28.3	**11**	11.517	11.4809	-0.517	-0.4809	0.036053
29	**11.2**	11.7116	11.6498	-0.5116	-0.4498	0.061754
30.2	**12**	12.0451	11.9393	-0.0451	0.06067	0.105811
31	**12.4**	12.2675	12.1323	0.13248	0.26766	0.1351821
27.5	**10.8**	11.2946	11.2879	-0.4946	-0.4879	0.006682
26.8	**10.2**	11.1	11.119	-0.9	-0.919	0.019018
29.6	**11.6**	11.8784	11.7946	-0.2784	-0.1946	0.083783
28.1	**10.8**	11.4614	11.4327	-0.6614	-0.6327	0.02871
30	**11.8**	11.9895	11.8911	-0.1895	-0.0911	0.098468
27.4	**10.2**	11.2668	11.2638	-1.0668	-1.0638	0.00301
28.8	**9.8**	11.656	11.6016	-1.856	-1.8016	0.05441
28.6	**10.3**	11.6004	11.5533	-1.3004	-1.2533	0.047067
31.2	**12.6**	12.3231	12.1806	0.27688	0.41941	0.142525
29.6	**11.6**	11.8784	11.7946	-0.2784	-0.1946	0.083783
28.1	**10.8**	11.4614	11.4327	-0.6614	-0.6327	0.02871
30	**11.8**	11.9895	11.8911	-0.1895	-0.0911	0.098468
23.3	**10.3**	10.1271	10.2746	0.17292	0.02541	0.1475181
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25.9	**9**	10.8498	10.9019	-1.8498	-1.9019	0.052061
25	**8.9**	10.5996	10.6847	-1.6996	-1.7847	0.085104
26.5	**10.5**	11.0166	11.0466	-0.5166	-0.5466	0.030033
20.3	**9.1**	9.29314	9.5508	-0.1931	-0.4508	0.257661
23.7	**10**	10.2383	10.3711	-0.2383	-0.3711	0.132832
21.9	**9.5**	9.7379	9.93682	-0.2379	-0.4368	0.198918
27.2	**10.7**	11.2112	11.2155	-0.5112	-0.5155	0.004333
28.1	**12.2**	11.4614	11.4327	0.73862	0.76733	0.0287106
26.7	**11.9**	11.0722	11.0949	0.82779	0.8051	0.0226895
27.4	**11.3**	11.2668	11.2638	0.03321	0.03622	0.0030106
29.1	**17.4**	11.7394	11.6739	5.66064	5.72606	0.0654249
27.5	**12.6**	11.2946	11.2879	1.30541	1.31209	0.006682
29	**12.8**	11.7116	11.6498	1.08844	1.15019	0.0617535
27.3	**11.2**	11.239	11.2397	-0.039	-0.0397	0.000661
21.9	**11.1**	9.7379	9.93682	1.3621	1.16318	0.1989181
23.7	**10.4**	10.2383	10.3711	0.16173	0.0289	0.1328324
22.8	**13.8**	9.98809	10.154	3.81191	3.64604	0.1658752
24.9	**11**	10.5718	10.6606	0.42816	0.33938	0.0887753
26.2	**12.3**	10.9332	10.9743	1.36678	1.32574	0.0410466
29	**12.7**	11.7116	11.6498	0.98844	1.05019	0.0617535
30.1	**10.9**	12.0173	11.9152	-1.1173	-1.0152	0.10214
30.6	**17.1**	12.1563	12.0358	4.94367	5.06417	0.1204964
34	**11.2**	13.1015	12.8561	-1.9015	-1.6561	0.245325
26.2	**12**	10.9332	10.9743	1.06678	1.02574	0.0410466
30.1	**11.6**	12.0173	11.9152	-0.4173	-0.3152	0.10214
29.5	**11.3**	11.8506	11.7704	-0.5506	-0.4704	0.080111
27.4	**10**	11.2668	11.2638	-1.2668	-1.2638	0.00301
25.9	**9.8**	10.8498	10.9019	-1.0498	-1.1019	0.052061
28.3	**11**	11.517	11.4809	-0.517	-0.4809	0.036053
29	**11.2**	11.7116	11.6498	-0.5116	-0.4498	0.061754
30.2	**12**	12.0451	11.9393	-0.0451	0.06067	0.105811
31	**12.4**	12.2675	12.1323	0.13248	0.26766	0.1351821
27.5	**10.8**	11.2946	11.2879	-0.4946	-0.4879	0.006682
26.8	**10.2**	11.1	11.119	-0.9	-0.919	0.019018
29.6	**11.6**	11.8784	11.7946	-0.2784	-0.1946	0.083783

28.1	**10.8**	11.4614	11.4327	-0.6614	-0.6327	0.02871
30	**11.8**	11.9895	11.8911	-0.1895	-0.0911	0.098468
27.4	**10.2**	11.2668	11.2638	-1.0668	-1.0638	0.00301
28.8	**9.8**	11.656	11.6016	-1.856	-1.8016	0.05441
28.6	**10.3**	11.6004	11.5533	-1.3004	-1.2533	0.047067
31.2	**12.6**	12.3231	12.1806	0.27688	0.41941	0.142525
29.6	**11.6**	11.8784	11.7946	-0.2784	-0.1946	0.083783
28.1	**10.8**	11.4614	11.4327	-0.6614	-0.6327	0.02871
30	**11.8**	11.9895	11.8911	-0.1895	-0.0911	0.098468

Figure 1 contains the absolute estimation errors for bull and cow for both procedures while Figure 2 is the between absolute estimation errors for both methods. In Figure 1, the $|d|$ is the absolute estimation errors between the bull horns width for the two procedures while $|d|1$ is the absolute estimation errors between the two procedures for the cow with horn. We see that that the variation in cow width horns for the two procedures is larger than the variation between the bull width horns measurement. In Figure 2, we observed that the estimation errors associated with the two methods, bull and cow width vary largely, this is due to the genetic balance of the gender of the cattle. We also observed that the cow width horns are larger than the bull width horns. The correlation of the estimated value for both procedures are perfect, that is one while the between absolute estimation error correlation is 0. 168. The correlation value of estimated and original data values for the two procedures, for the bull and cow horns width are 0.46 and 0.45 respectively. This implies that the correlation values of the estimated values and the original values are weakly positive. The correlation value computed using the estimated values for the two techniques revealed that there is perfect positive correlation, implying that the linear estimation and the least square estimation performance is similar.

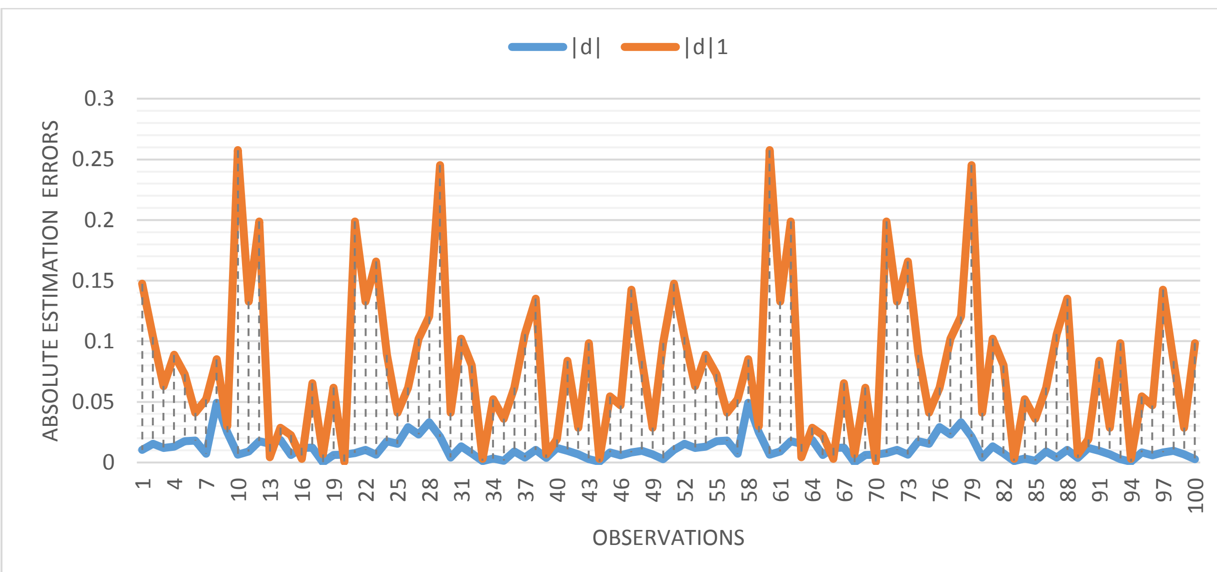

Figure 1: Comparative analysis between estimation errors difference $|d|$

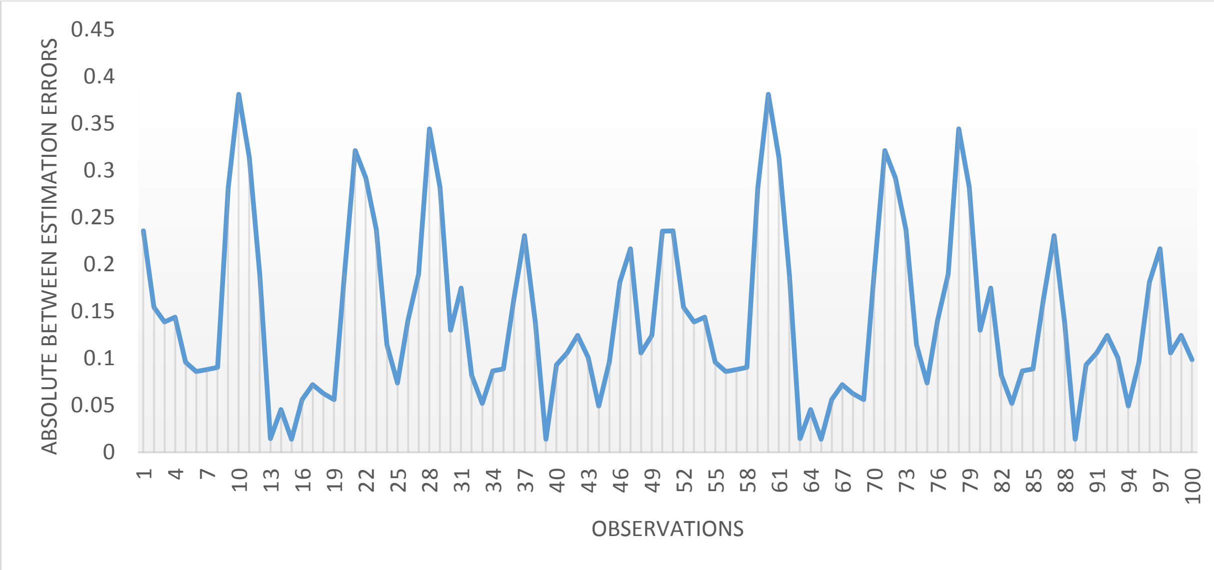

Figure 2: Absolute between estimation error

The power of estimation for the estimated values for both procedures is 0.5, indicating that both procedures have similar estimated values for the data set. It also showed that these two methods are reliable and stable to perform estimation. The correlation between the observed values and original data for both techniques is though positively weak. That is, the correlation is 0.46 *and* 0.45 respectively. The power of prediction between the original data and the estimated data is $P_{ca} = 0.732$, and $P_{ca} = 0.739$ indicating that the power of estimation is robust with weak $\rho_{xy} = 0.46$ and $\rho_{xy} = 0.45$ for the bull and cow horn width respectively.

4.0. Conclusions

The two estimation procedures based on the cattle data set revealed that both procedures are very suitable to perform estimation for a given data set. The analysis showed that the correlation between the two procedures are perfectly correlated and the correlation associated to each of the procedures for the data set revealed weak positive correlation. The power of estimation demonstrated that both procedures have robust estimation showing that each procedure has equal chances of estimating the given data suitably. Therefore, this outcome revealed that these procedures are suitable, reliable, and stable to perform estimation for any data set.

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