Supported by tetfund

Numerical Analysis of Third grade nanofluid with convective boundary conditions in the presence of heat absorption and thermal radiation

Aroloye Soluade Joseph[1], Fenuga Olugbenga John[2], Abiala Israel Olutunji[3], Okoya Samuel Segun[4]
[1,2,3]Department of Mathematics, Faculty of Science, University of Lagos.
[4]Department of Mathematics, Faculty of Science, Obafemi Awolowo University Ife.
Corresponding author email: saroloye@unilag.edu.ng

Received: September 14, 2024 Accepted: November 28, 2024

Abstract: Numerical solutions of third grade nanofluid in the presence of viscous dissipation, heat absorption, magnetic effect, thermal radiation, and convective boundary conditions is investigated. Influence of thermophoresis and Brownian motion are also considered in the problem. The similarity solution is used to transform the system of partial differential equations, describing the problem under consideration, into a boundary value problem of coupled ordinary differential equations, and an efficient numerical technique is implemented to solve the reduced system. The results are presented graphically and in tabular form and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters. Results of temperature and nanoparticle concentration are plotted and discussed for various values of material parameters, Prandtl number, Lewis number, Newtonian heating parameter, Eckert number, thermophoresis and Brownian motion parameters. Numerical computations are performed. The results show that the change in temperature and nanoparticle concentration distribution functions is similar when bigger values of material parameters β_1 and β_2 are used. The results further revealed that the temperature and thermal boundary layer thickness are increasing functions of Newtonian heating parameter γ. An increase in thermophoresis and Brownian motion parameters lead to an enhancement in the temperature. The results are compared with existing results in literature and there is excellent agreement.

Key words: convective boundary conditions, nanofluid, Numerical, thermal radiation, third grade and viscous dissipation,

Introduction

The interest in electronic technologies has been increased fastly, especially in miniaturization of the computing and communication devices. Thermal performance of such devices is a challenging problem for the engineers and the investigators. A new variety of heat transfer fluids is introduced, namely the nanofluids. The nanoscale solid particles are added in the base fluid. Such additive technology is employed to enhance thermal characteristics of base fluids. In fact, the thermal performance of ordinary base fluids is not suitable to meet the cooling requirements in the industrial processes. The novel characteristics of nanofluids make them strongly applicable in different processes of heat transfer. Examples of such processes are fuel cells, microelectronics, hybrid-powered engines, pharmaceutical applications, etc. Naturally, the dispersion of nanoparticles and an increase in the thermal conductivity bring new and additional ideas for the investigators and engineers that can be used in heat transfer applications. Examples of such applications include automotives, refrigeration, chemical industry, food processing industry, petroleum industry, etc. Motivated by such facts, Choi 1995) discovered in his investigation that the presence of nanoparticles in base fluid increases thermal characteristics. Mathematical analysis of nanofluids with thermophoresis and Brownian motion effects was studied by Buongiorno (2006). Multiscale properties of multi component flow of nanofluid and method were examined by Zhou *et al.* (2010). Makinde and Aziz (2010) investigated the convective thermal condition effect in boundary layer flow of viscous nanofluid over a stretching sheet. Turkyilmazoglu (2013) analyzed the unsteady flow of viscous fluid past a vertical flat plate in the presence of different types of nanoparticles. Second law analysis in steady flow of magneto-nanofluid induced by a porous disk was examined by Rashidi *et al* (2013). Turkyilmazoglu and Pop (2013) explored the properties of heat and mass transfer in unsteady natural convection flow of viscous nanofluid in the presence of thermal radiation effect. Analytical treatment of magneto hydrodynamic flow of nanofluid in a porous channel was provided by Sheikholeslami *et al.* (2013). Mustafa *et al* (2013) numerically investigated the two-dimensional stagnation point flow of nanofluid due to an exponentially stretching sheet. Ibrahim and Makinde (2013) examined the effects of thermal and concentration stratification in mixed convection flow of nanofluid past a vertical flat plate. Rotating flow of nanofluid in the presence of an applied magnetic field was examined by Sheikholeslami *et al.* (2014). Hayat *et al.* (2014) analyzed the effect of convective heat and mass conditions in peristaltic flow of nanofluid.

Flows of non-Newtonian fluids are quite prominent in many industrial and engineering processes. There are certain materials like shampoos, muds, soaps, apple sauce, sugar solution, polymeric liquids, tomato paste, condensed milk, paints, blood at low shear rate, which show the characteristics of non-Newtonian fluids. The behavior of such materials cannot be explored by a single constitutive relationship because of their diverse properties. Hence, different fluid models are developed in the past to describe the exact nature of non-Newtonian materials. The fluid model under consideration is a subclass of differential type non-Newtonian fluid namely the third grade. The third grade fluid model exhibits shear thickening and shear thinning characteristics. Abelman *et al*. (2009) investigated Couette flow of a third grade fluid with rotating frame and slip condition. Sajid *et al*. (2008) investigated Finite element solution for flow of a third grade fluid past a horizontal porous plate with partial slip. Sahoo and Do (2010) examined Effects of slip on sheet-driven flow and heat transfer of a third grade fluid past a stretching sheet. Makinde and, Chinyoka (2011) investigated Numerical study of unsteady hydromagnetic Generalized Couette flow of a reactive third-grade fluid with asymmetric convective cooling. Abbasbandy and Hayat (2011) examined On series solution for unsteady boundary layer equations in a special third grade fluid. Hayat and Abbasi (2011) investigated Variable viscosity effects on the peristaltic motion of a third-order fluid. Aziz and Mahome (2013) studied Reductions and solutions for the unsteady flow of a fourth grade fluid on a porous plate. Hatami *et al*. (2014) Computer simulation of MHD blood conveying gold nanoparticles as a third grade non-Newtonian nanofluid in a hollow porous vessel

Muhammad et al. (2024). Thermal dynamics of nanoparticle aggregation in MHD dissipative nanofluid flow within a wavy channel: Entropy generation minimization. Muhammad Awais (2024) investigated thermal dynamic of stagnated flow of MHD Jeffery fluid when the joule heating, viscous dissipation and Soret effect are present: A multistep Milne's approach. Aroloye Soluade Joseph and Owa David Oluwarotimi.(2024) carried out theoretical investigation on Numerical Solution of Drug Diffusion Model using the Classic Runge Kutta method (2024). Theboundary layer flow of non-Newtonian fluid generated by a moving surface has great interest in the industrial and technological applications. Such applications include copper wires, polymer extrusion, glass fiber, paper production, crystal growing, manufacture of plastic sheets, drawing of plastic films and wires, and the boundary layer along a liquid film condensation process, etc. In addition, the simultaneous effects of heat and mass transfer in boundary layer flow of non-Newtonian fluids are much more important in heat exchange for sensible heat storage beds, electrochemical processes, insulation of nuclear reactors, petroleum reservoirs, high performance chemical catalytic reactors and many others.

The aim here is to explore the characteristics of nanoparticles in boundary layer flow of the third grade fluid in the presence of viscous dissipation, chemical reaction, heat absorption and thermal radiation subjected to convective boundary conditions. Effects of thermophoresis and Brownian motion are also incorporated into the investigation. Newtonian thermal condition is utilized for heat transfer analysis. Mathematical modelling is performed under boundary layer assumptions. Similarity variables are employed to convert the partial differential equations into the ordinary differential equations. Numerical analysis method via shooting method with six order Runge Kutta scheme is explored to provide numerical solutions to dimensionless velocity, temperature and concentration models. Graphs and tables are presented to examine the impacts of physical parameters on the temperature and concentration fields. Liao (2012), Turkyilmazoglu(2012), Rashidi *et al*. (2012), Shehzad *et al*.(2013), Abbasbandy *et al.* (2013), Hayat(2013) and Shehzad *et al*.(2014) have all eployed analytical approach to solve similar problems

Methodology

We consider the two-dimensional incompressible flow of the third grade fluid generated by a stretching surface in the presence of viscous dissipation, thermal radiation magnetic effect and convective boundary conditions. The sheet is stretched with the velocity $u_w(x) = cx,$ where c denotes a constant. Momentum, heat and mass transfer characteristics are considered in the presence of thermophoresis, Brownian motion, Newtonian heating, heat absorption and magnetic effects. The considered flow is hydrodynamic due to which the influence of Joule heating is not taken into account. Following Sahoo and Do (2010), Hayat et al. (2014), and Shehzad *et al.* (2015), governing boundary layer equations for third grade the nanofluid with viscous dissipation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left(u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + 3\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} \right) + 2\frac{\alpha_2}{\rho}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\alpha_3}{\rho}\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + r\left(D_B \frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 \right) + \frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\alpha_1}{\rho c_p}\left(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} \right) + 2\frac{\alpha_3}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^4 + \frac{Q_0}{(\rho C_p)}(T - T_\infty) - \frac{1}{(\rho C_p)} q_r \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-Kr\left(C-C_\infty\right) \quad (4)$$

The appropriate boundary conditions for the present flow problems are

$$u=u_w(x)=cx, v=0, \frac{\partial T}{\partial x}=h_s T, C=C_w \quad at\ y=0 \quad (5)$$

$$u\to 0, v\to 0, T\to T_\infty, C\to C_\infty \quad as\ y\to\infty \quad (6)$$

where u and v denote the velocity components parallel to the *x*- and *y*-directions, α_1,α_2 and α_3 are the material parameters, $v=\left(\mu/\rho\right)$ is the kinematic viscosity, u is the dynamic viscosity, ρ is the density of fluid, *T* is the fluid temperature, α is the thermal diffusivity, $r=(\rho c)_p/(\rho c)_f$ is the ratio of nanoparticle heat capacity to the base fluid heat capacity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, *C* is the concentration, c_p is the specific heat at constant pressure, T_∞ and C_∞ are the ambient temperature and concentration away from the sheet and h_s is the heat transfer coefficient, kr is the constant rate of chemical reaction, Q_0 is the coefficient of internal heat generation, q_r is the radiative heat flux. Letting

$$u=cxf'(\eta), y=-\sqrt{cvf(\eta)}, \eta=y\sqrt{\frac{c}{v}}, \quad \theta(\eta)=\frac{T-T_\infty}{T_\infty}, \phi(\eta)=\frac{C-C_\infty}{C_\infty} \quad (7)$$

Eq. (2) is satisfied automatically and Eqs. (3)-(6) take the forms:

$$f'''+ff''-f'^2+\beta_1\left(2ff''-ff'''\right)+\left(3\beta_1+2\beta_2\right)f''^2+6\varepsilon_1\varepsilon_2 f'''f''^2=0 \quad (8)$$

$$\theta''+\Pr f\theta'+prEcf''^2+\Pr Ec\beta_1 ff''^2-\Pr Ec\beta_1 ff'f'''+2\Pr Ec\varepsilon_1\varepsilon_2 f''^4+\Pr N_b\theta'\phi'+\Pr N_t\theta'^2-Q\theta+Ra\theta''=0 \quad (9)$$

$$\phi''+\Pr Lef\phi'+\left(N_t/N_b\right)\theta''-Kc\phi=0 \quad (10)$$

$$f(0)=0, f'(0)=1, \theta'=-\gamma\left(1+\theta(0)\right)=0 \quad (11)$$

$$f'(\infty)\to 0, \theta(\infty)\to 0, \phi(\infty)\to 0 \quad (12)$$

Where $\beta_1=\frac{c\alpha_1}{\mu}, \beta_2=\frac{c\alpha_2}{\mu}, \varepsilon_1=\frac{c\alpha_3}{\mu}$ are the material parameters for third grade fluid, $\varepsilon_2=\frac{cx^2}{v}$ is the local Reynolds number, $\Pr=\frac{v}{\alpha}$ is the Prandtl number, $Ec=\frac{u_w^2}{\left(c_p T_\infty\right)}$ is the Ecket number, $N_b=\frac{(\rho c)_p D_B\left(C_w-C_\infty\right)}{(\rho c)_f v}$ is the Brownian motion parameter, $N_t=\frac{(\rho c)_p D_T}{(\rho c)_f v}$ is the thermophoresis parameter and $Le=\frac{\alpha}{D_B}$ is the Lewis number, $Q=\frac{v^2 Q_0}{\alpha\rho C_p}$ is the heat source parameter, $Kc=\frac{vKr}{U_0^2}$ is the chemical reactions, $Ra=\frac{16\alpha T_\infty^3}{3k^*\alpha\rho_f C_p}$ is the thermal radiations.

The dimensionless expressions of skin-friction coefficient, local Nusselt and Sherwood numbers can be written as follows:

$$Cf_x \mathrm{Re}_x^{1/2}=\left(f''+\beta_1\left(3ff''-ff'''\right)+2\varepsilon_1\varepsilon_2 f''^3\right)_{\eta=0}$$

$$Nu\,\mathrm{Re}_x^{-1/2}=1+\frac{1}{\theta(0)}, Sh\mathrm{Re}_{x-1/2}=-\phi'(0)$$

The system of highly non-linear differential equations (8), (9) and (10) subjected to boundary conditions (11) and (12) are solved by a numerical approach via shooting method with the six-order Runge-Kutta method for different moderate values of the flow, heat and mass transfer parameters. The effective Broyden technique is adopted in order to improve the initial guesses and to satisfy the boundary conditions at infinity. Maple software is used to code and simulate the above numerical procedure

Numerical Result and discussion

Table1: Convergence of numerical solution for different order of approximate when $\beta_1 = \beta_2 = 0.1, \varepsilon_1 = 0.2, \Pr = 1.2, Le = 0.8, \gamma = 0.1, Nt = 0.1, Nb = 0.2, Ec = 0.5, h_f = -0.6$ *and* $h_\theta = h_\phi = -0.8, Kc = 0, Q = 0, Ra = 0$

	$-f''(0)$		$-\theta'(0)$			
Order of Approximation	Shehzad *et al.* (2015)	Present Result	Shehzad *et al.* (2015)	Present Result	Shehzad *et al.* (2015)	Present Result
1	0.81400	0.81400	0.13497	0.13497	0.63911	0.63911
5	0.79768	0.79768	0.15866	0.15866	0.56813	0.56813
12	0.79791	0.79791	0.16119	0.16119	0.48367	0.48367
20	0.79791	0.79791	0.16113	0.16113	0.48106	0.48106
28	0.79791	0.79791	0.16113	0.16113	0.48088	0.48088
35	0.79791	0.79791	0.16113	0.16113	0.48088	0.48088
40	0.79791	0.79791	0.16113	0.16113	0.48088	0.48088

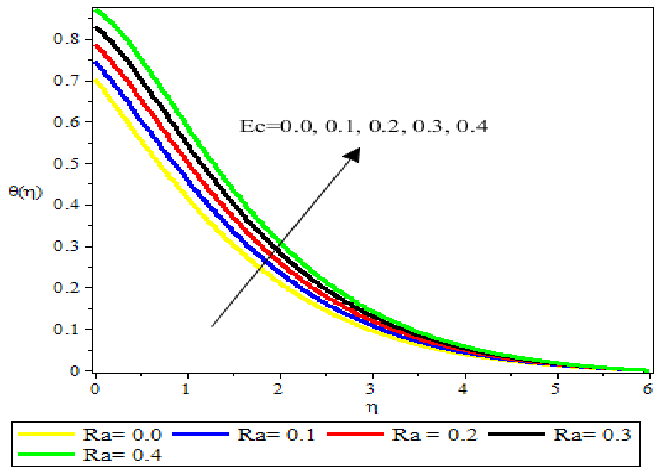

Fig. 1 Influence of *Ra* on temperature $\theta(\eta)$

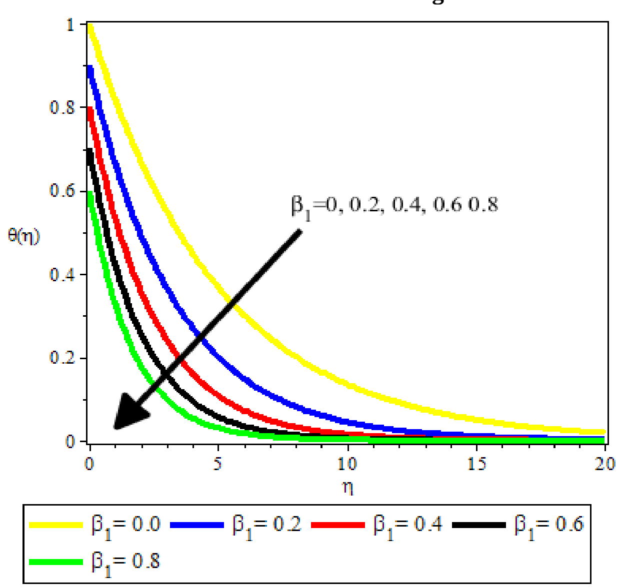

Fig. 2 Influence of β_1 on temperature $\theta(\eta)$

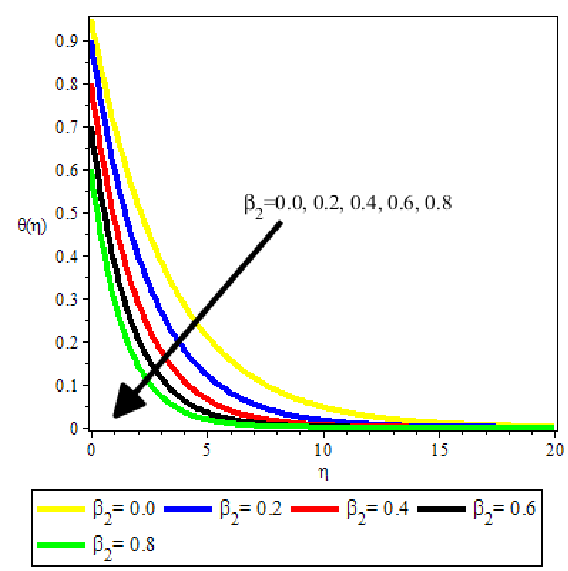

Fig. 3 Influence of $\beta2$ on temperature $\theta(\eta)$

We plot the solutions of dimensionless temperature $\theta(\eta)$ and nanoparticle concentration $\phi(\eta)$ for the multiple values of material parameters β_1 and β_2, Prandtl number Pr, Lewis number Le, Newtonian heating parameter γ, thermophoresis and Brownian motion parameters Nt and Nb and Eckert number Ec. Figure 1 show the dimensionless temperature with various value of radiation parameter. It is seen from the figure that fluid temperature increases as thermal radiation parameter increases due to the energy supply that heat up the fluid

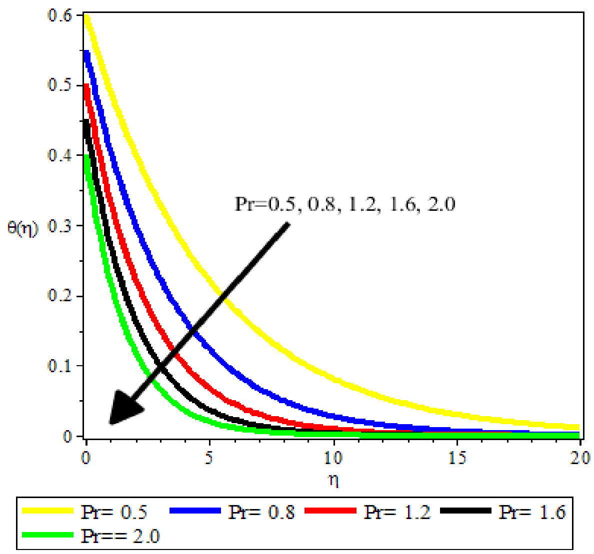

Fig. 4 Influence of Pr on temperature $\theta(\eta)$

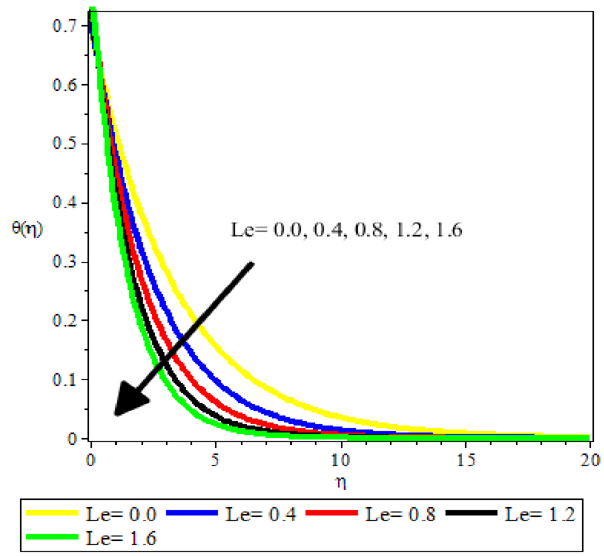

Fig. 5 Influence of Le on temperature $\theta(\eta)$

Figures $2-9$ are sketched to examine the temperature distribution function $\theta(\eta)$ corresponding to different values of β_1, β_2, Pr, Le, γ, Nt, Nb and Ec. Figures 2 and 3 present that an increase in material parameters tends to a decrease in the temperature and thermal boundary layer thickness. Here, the material parameters depend on normal stresses and viscous forces. Normal stresses are increased and viscous forces are decreased when we increase the values of material parameters. This change in normal stresses and viscous forces tends to a reduction in the temperature and thermal boundary layer thickness. An increase in Prandtl number Pr shows a decrease in temperature and thermal boundary layer thickness (see Fig. 4). Prandtl number is inversely proportional to the thermal diffusivity of fluid. Thermal diffusivity is weaker for higher Prandtl fluids and stronger for lower Prandtl fluids. Weaker thermal diffusivity corresponds to lower temperature and stronger thermal diffusivity shows higher temperature. Here, thermal diffusivity is responsible for change in temperature.

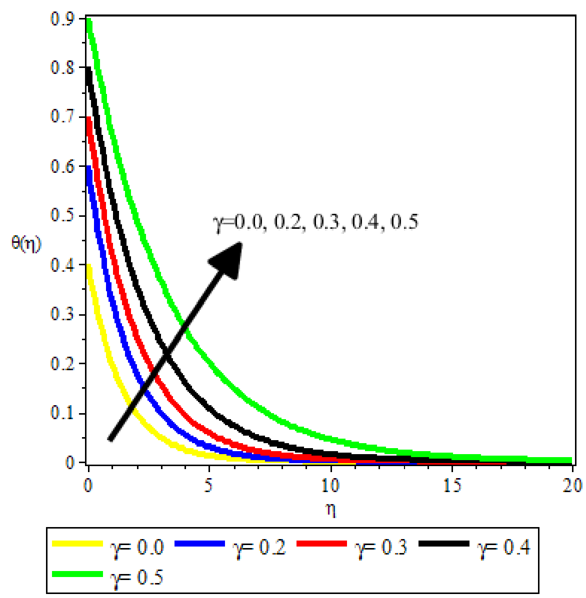

Fig. 6 Influence of γ on temperature $\theta(\eta)$

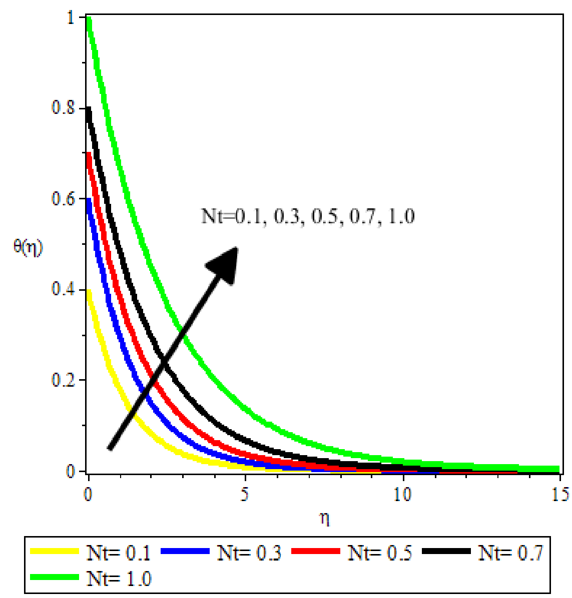

Fig. 7 Influence of Nt on temperature $\theta(\eta)$

Figure 5 clearly shows that an increase in Lewis number leads to a reduction in the temperature and thermal boundary layer thickness. Lewis number involves the diffusion coefficient. Increasing values of Lewis number corresponds to decrease in diffusion coefficient. This smaller diffusion coefficient tends to a lower temperature. Effects of Newtonian heating parameter on the temperature are examined in Fig. 6. Temperature is increased when we increase the values of Newtonian heating parameter. It is also seen that the temperature at the wall is an increasing function of Newtonian heating parameter. Newtonian heating parameter is directly proportional to the conjugate heat transfer coefficient. Conjugate heat transfer coefficient increases when we increase the Newtonian heating parameter due to which the temperature rises. It is obvious from Figs. 7 and 8 that temperature and thermal boundary layer thickness are enhanced when we use higher values of thermophoresis and Brownian motion parameters. Further, we examine that the temperature at the wall for $Nb = 1.0$ is slightly greater than $Nt = 1.0$. Figure 9 depicts that the temperature and thermal boundary layer thickness are enhanced when we increase the values of Eckert number Ec.

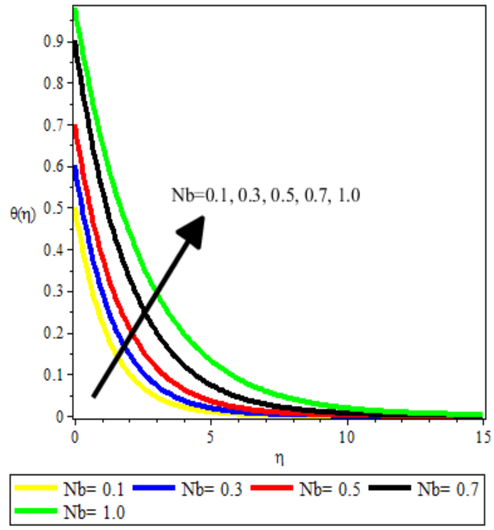

Fig. 8 Influence of *Ec* on temperature $\theta(\eta)$

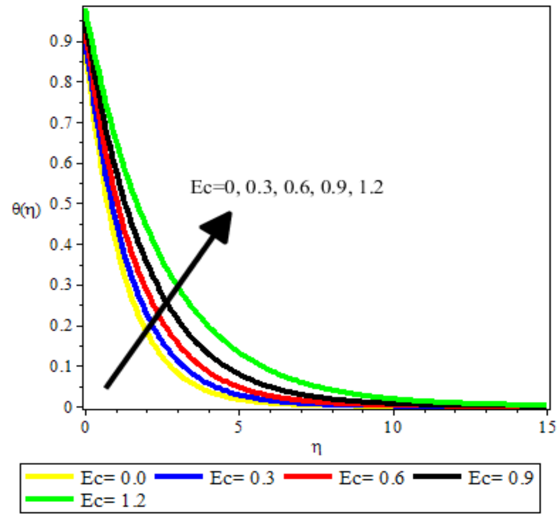

Fig. 9 Influence of *Nb* on temperature $\theta(\eta)$

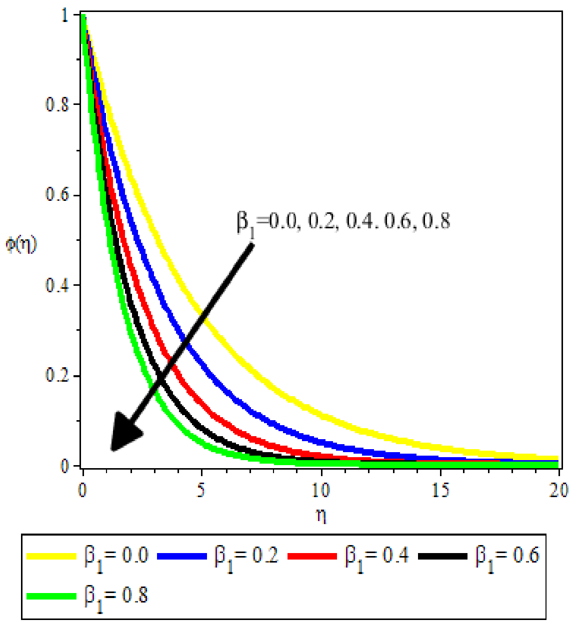

Fig. 10 Influence of $\beta 1$ on concentration $\phi(\eta)$

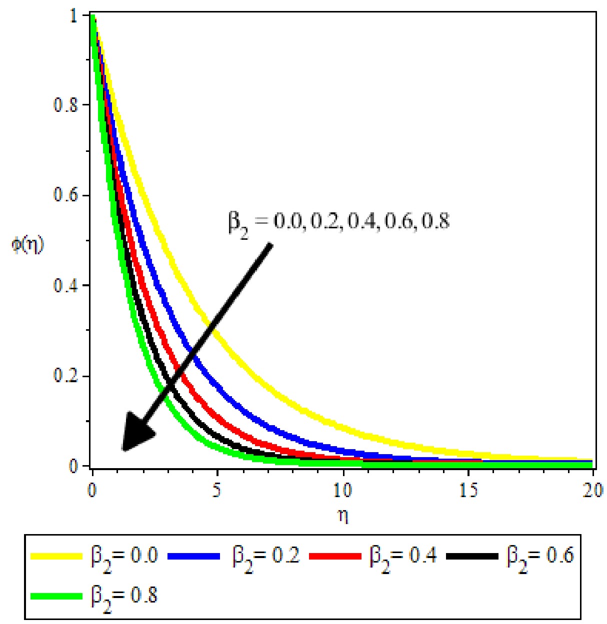

Fig. 11 Influence of $\beta 2$ on nanoparticle concentration $\phi(\eta)$

To analyze the variations in nanoparticle concentration distribution function $\phi(\eta)$ for various values of material parameter β_1 and β_2, Prandtl number Pr, Lewis number Le, thermophoresis and Brownian motion parameters Nt and Nb and Eckert number Ec, we have drawn Figs. 10−16. From Figs. 10 and 11, we observe that the nanoparticle concentration and its related boundary layer thickness are lower for higher values of material parameters. We note that the material parameters have similar trends for temperature and nanoparticle concentration but the reduction in temperature is more pronounced in comparison to nanoparticle concentration. Higher values of Prandtl number tends to a decrease in the nanoparticle concentration and boundary layer thickness (see Fig. 12). Figure 13 indicates that an increase in Lewis number leads to a weaker nanoparticle concentration and its associated boundary layer thickness. A comparison of Figs. 7 and 14 shows that temperature and nanoparticle concentration fields are increasing functions of thermophoresis parameter Nt. From Fig. 15, it is examined that an increase in the values of Brownian motion parameter creates a reduction in the nanoparticle

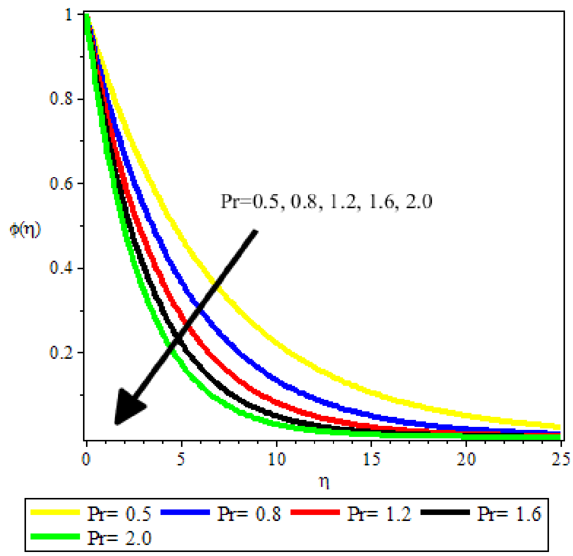

Fig. 12 Influence of *Pr* on concentration f $\phi(\eta)$

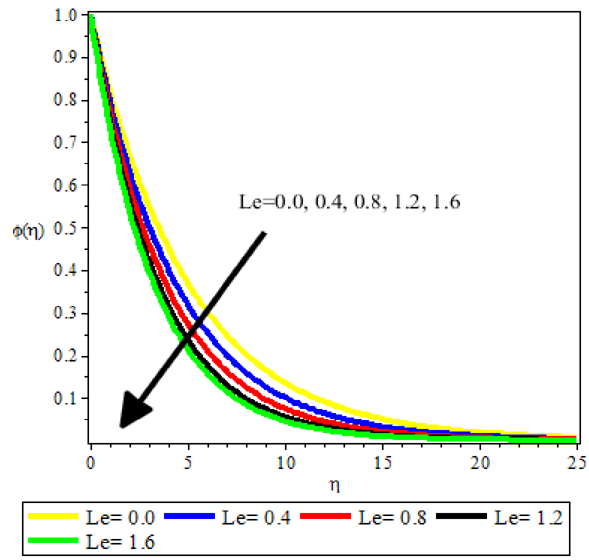

Fig.13 Influence of *Le* concentration $\phi(\eta)$

concentration and boundary layer thickness. The concentration is an increasing function of Eckert number (see Fig. 16). The values of $Cf_x \operatorname{Re}_x^{1/2}$ are decreased by increasing β_1, ε_1 and ε_2, as given in Table 2.

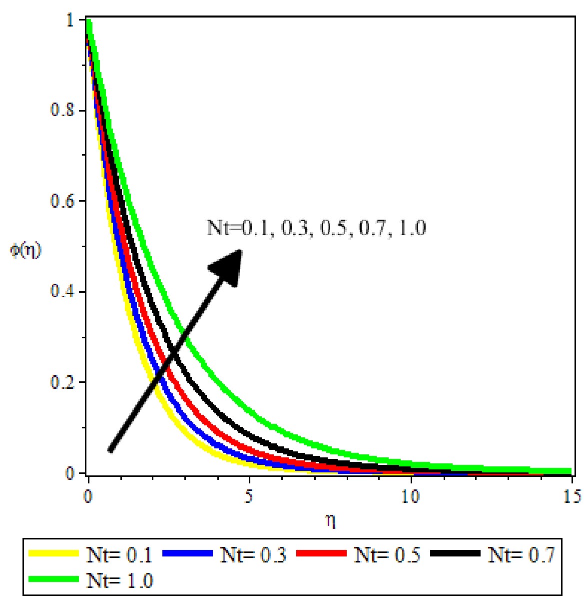

Fig. 14 Influence of *Nt* on concentration $\phi(\eta)$

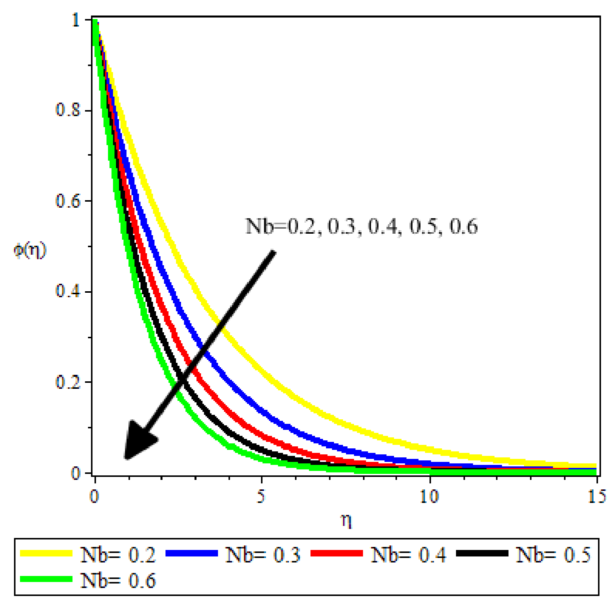

Fig.15 Influence of *Nb* on concentration $\phi(\eta)$

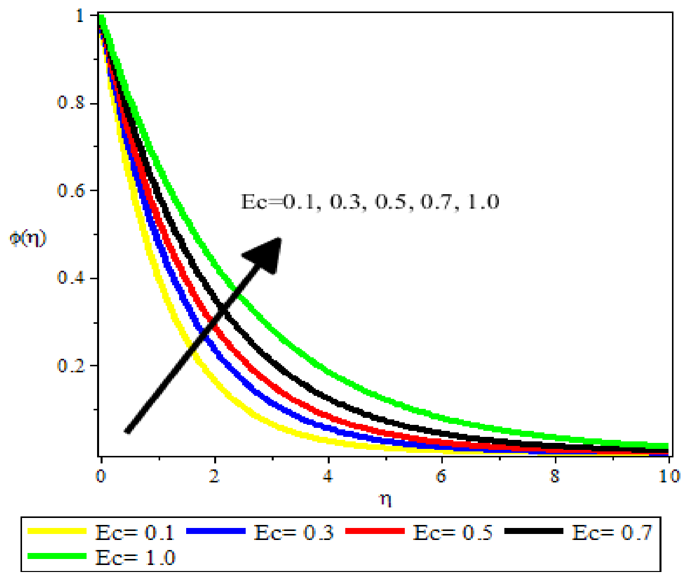

Fig. 16 Influence of *Ec* on nanoparticle concentration $\phi(\eta)$

Table 2: Numerical Value of skin-friction coefficient $Cf_x \operatorname{Re}_x^{1/2}$ for different values of $\beta_1, \beta_2, \varepsilon_1$ *and* ε_2

				$Cf_x \operatorname{Re}_x^{1/2}$	
β_1	β_2	ε_1	ε_2	Shehzad *et al.* (2015)	Present study
0	0.1	0.1	0.2	0.92032	0.92032
0.2	0.1	0.1	0.2	1.16531	1.16531
0.3	0.1	0.1	0.2	1.26729	1.26729
0.1	**0**	0.1	0.2	**1.10212**	**1.10212**
0.1	**0.2**	0.1	0.2	**1.00542**	**1.00542**
0.1	**0.2**	0.1	0.2	**0.96309**	**0.96309**
0.1	0.1	0	0.2	1.06291	1.06291
0.1	0.1	0.3	0.2	1.03190	1.03190
0.1	0.1	0.5	0.2	1,01504	1,01504
0.1	0.1	**0.2**	**0**	**1.06291**	**1.06291**
0.1	0.1	**0.2**	**0.3**	**1.03190**	**1.03190**
0.1	0.1	**0.2**	**0.5**	**1.01504**	**1.01504**

Conclusion

We examine that the change in temperature and nanoparticle concentration distribution functions is similar when we use higher values of material parameters β_1 and β_2. It is seen that temperature and thermal boundary layer thickness are increasing functions of Newtonian heating parameter γ. An increase in thermophoresis and Brownian motion parameters tends to an enhancement in the temperature. Temperature and nanoparticle concentration are enhanced for larger values of Eckert number Ec. The effects of Eckert number on temperature are more pronounced in comparison with the nanoparticle concentration. The nanoparticle concentration is decreased rapidly for smaller values of Brownian motion parameter but this change is very slow when $Nb > 0.4$.

Reference

Abbasbandy S, and Hayat, T .(2011). On series solution for unsteady boundarylayer equations in a special thirdgrade fluid [J]. Communications in Nonlinear Science and Numerical Simulation, 16: 3140−3146.

Abbasbandy, S. Hashemi, M.S. and Hashim M. On convergence of homotopy analysis method and it s application to fractionalintegro-differential equations.*Quaestiones Mathematicae* ,2013, 36: 93−105.

Abelman, S. Momoniat E, and Hayat T. (2009).Couette flow of a third grade fluid with rotating frame and slip condition. *Nonlinear Analysis: Real World Applications* 10: 3329−3334.

Aroloye Soluade Joseph and Owa David Oluwarotimi. (2024).Numerical Solution of Drug Diffusion M odel using the Classic Runge Kutta method (2024). Journal of Nigerian Association of Mathematical Physics . 8, 2 (2024) 117-128

Aziz, T. and Mahomed, F. M. (2013). Reductions and solutions for the unsteady flow of a fourth grade fluid on a porous plate. *Applied Mathematics and Computation*, 219: 9187−9195.

Buongiorno J. (2010).Convective transport in nanofluids. *Journal of Heat Transfer*, 2006, 128: 240−250.

Choi Sus.(1995). Enhancing thermal conductivity of fluids with nanoparticles. *ASME MD* 231, 66: 99−105.

Hatami, M. Hatami J, and Ganji D. D. (2014,) Computer simulation of MHD blood conveying gold nanopa r ticles as a third grade non-Newtonian nanofluid in a hollow porous vessel. *Computers Methods Program in Biomedicine*, 113: 632−641.

Hayat T, and Abbasi F. M. Variable viscosity effects on the peristaltic motion of a third order fluid *Intern ati onal Journal of Numerical Methods in Fluids*, 67: 1500−1515.

Hayat T, Shehzad S A, and Alsaedi A.(2013) vThree-dimensional radiative flow with variable thermal conductiv ity in a porous medium. *European Physical Journal Plus*, 128: 67.

Hayat T, Shehzad S A, Qasim M, Asghar S, and Alsaedi, A. (2014). Thermally stratified radiative flow of third grade fluid over a stretchi ng surface. *Journal of Thermophysics & Heat Transfer*, 28: 155−161.

Hayat T, Yasmin H, Ahmad B, and Chen B. (2014) Simultaneous effects of convective conditi ons and nanoparticles on peristaltic motion. *Jour nal of Molecular Liquids*, 193: 74−82.

Ibrahim, W, and Makinde, O.D.(2013).The effect of double stratification on boundary layer flow and heat transfer of nanofluid over a vertical plate *Computers & Fluids*, 86: 433−441.

Liao, S.J. (2012.)Homotopy analysis method in nonlinear differential equations. Heidelberg: Springer & Higher Education Press,

Makinde O D, and Chinyoka T. (2011). Numerical study of unsteady hydromagnetic Generalized Couette flow of a reactive third-grade fluid with asymmetric convective cooling. *Computers Mathematicswith Applications*, 61: 1167−1179.

Makinde, O. D and Aziz, A. (2010).MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition. *International of Journal Thermal Sciences*, 49: 1813−1820.

Muhammad Idrees Afridi, Abid Hussanan, Muhammad Qas im, and Ali J. Chamkha (2024). Thermal dynamics of nanoparticle aggregation in MHD dissipative nanofluid flow within a wavy channel: Entropy generation minimization. Case Studies in Thermal Engineering. Volume 61, September 2024, 105054

Muhammad Awais, Salahuddin T., Maawiya Ould Sidi, Af nan Al Agha, and Hakim Al Garalleh.(2024) Insight into thermal dynamic of stagnated flow of MHD Jeffery fluid when the joule heating, viscous dissipation and Soret effect are present: A multistep Milne's approach. Case Studies in Thermal Engineering. Volume 64, December 2024, 105506

Mustafa M, Farooq, M A, Hayat T and Alsaedi, A. (2013). Numerical and series solutions for stagnation poi nt flow of nanofluid over an exponentia lly stretching sheet [J]. Plos One, 8: 61859.

Rashidi M M, Abelman S, Mehr N F. (2013). Entropy generation in steady MHD flow due to a rotating porous disk in a nanofluid .*International Journal of Heat andMass Transfer*, 62: 515−525.

Rashidi, M. M, Momoniat, E. and Rostami, B.(2012). Analytic approximate solut ions for MHD boundary-layer viscoelastic fluid flow over continuously moving stretching surface by homoto py analysis method with two auxiliary parameters. *Journal of Applied Mathematics*,780415.

Sahoo, B. and Do, Y. Effects of slip on sheet-driven flow and heat transfer of a third grade fluid past a stretching sheet. *International Communicat ions in Heat and Mass Transfer*, 2010, 37: 1064−1071.

Sajid M, Mahmood R, and Hayat T. (2008). Finite element solution for flow of a third grade fluid past a horizontal porous plate with partial slip. *Computers Mathematics with Applications*, 56: 1236−1244.

Shehzad, S A, Alsaadi F E, Monaquel S J, Hayat T.(2013). Soret and Dufour effects on the stagnation point flow of Jeffery fluid with convective boundary conditions. *European Physical Journal Plus*, 2013, 128: 56.

Shehzad, S A. Hayat, T. Alhuthali M S, Asghar S. MHD three-dimensional flow of Jeffrey fluid with Newtonian heating. *Journal of Central South University*, 2014, 21: 1428−1433.

Shehzad, S. A. Tariq Hussai, Hayat, T. Ramzan, M. and Alsaedi, A. (2015). Boundary layer flow of third grade nanofluid with Newtonian heating and viscous dissipation, *J. Cent. South Univ*. 22: 360−36

Sheikholeslami M, Hatami M, Ganji D D. (2013).Analytical investigation of MHD nanofluid flow in a semi- porous channel *Powder Technology*, 246: 327−336.

Sheikholeslami M, Hatami M, Ganji D D. (2014).Nanofluid flow and heat transfer in a rotating system in the presence of a magnetic field . *Journal of Molecular Liquids*, 190: 112−120.

Turkyilmazoglu M, Pop, I. (2013). Heat and mass transfer of unsteady natural convection flow of some nanofluids past a vertical infinite flat plate with radiation effect. *International Journal of Heat and Mass Transfer*, 59: 167−171.

Turkyilmazoglu M. (2013). Unsteady convection flow of some nanofluids past a moving vertical flat plate with heat transfer. *Journal of Heat Transfer*, 136: 031704.

Turkyilmazoglu, M.(2012) Solution of the Thomas-Fermi equation with aconvergent approach. *Com munications in Nonlinear Science and Numerical Simulation*, 17: 4097−4103.

Zhou L, Xuan Y, Li, Q. Multiscale simulation of flow and heat transfer of nanofluid with lattice Boltzmann method. *International Journal of Multiphase Flow*, 36: 364−374